Journal of Research in Science, Mathematics and Technology Education

https://jrsmte.com/

Knight or Knave? Description and Evaluation of a Programme for the Introduction of Logic at Primary School

Luigi Bernardi

Universidad Distrital Francisco José de Caldas, Colombia

Riccardo Manghi

Luiss Guido Carli, Italy

Abstract: The purpose of this work is to present and analyse an educational programme for primary school focused on the study of logic. The programme introduces logic through embodied cognition, linguistic reasoning, awareness of mistakes, and symbolic representation. Our core research question examines whether early exposure to logic supports the development of rational thinking, thus justifying its inclusion from the early stages of education. Although primarily qualitative, the analysis is complemented by quantitative methods, which serve as additional tools for verification and validation.

Keywords: Knights and knaves; Logic; Mathematics education; Primary school; Semiotic.
DOI: https://doi.org/10.31756/jrsmte.418SI

Introduction

Logic lies at the heart of mathematical and scientific thinking, and is fundamentally linked to certain elements of language. According to Ferrari and Gerla (2015), the constant attention paid to mathematical language, to the distinction between language and metalanguage, and to the notion of interpretation when working with logic makes it a tool suitable for teaching and learning at every educational stage. However, as pointed out by Durand-Guerrier et al. (2012), the educational role of logic is not always recognised. There may be several reasons for this: on the one hand, formal logic can be seen as an unnecessary tool that risks complicating teaching practice; on the other, some believe that basic logical abilities are developed irrespective of a targeted theoretical treatment. For example, the concept of 'not' is "considered as a very simple notion […] that does not need to be taught or discussed at this level [primary school]" (Durand-Guerrier, 2021, p. 6). But, according to Durand-Guerrier, the lack of an explicit treatment creates difficulties in understanding negation that can persist until university level, such as the fact that the connection between the negation of a universal statement and the role of counterexamples is never fully clarified. Another reason for the omission of logic in the school curriculum is that teaching of mathematics tends to prioritise exercises that develop the skills required for improving grades and meeting school targets; hence, any topics that are not deemed essential for future study—even if considered important for individual development—are sometimes left to one side. In our opinion, the idea that an understanding of basic logical concepts can be acquired automatically through standard mathematical teaching is wishful thinking. Nevertheless, we agree with some of the criticisms levelled at ways of teaching logic in which syntax is favoured over semantics.

The mistrust towards logic is majorly highlighted when dealing with symbolism. Returning to Durand-Guerrier (2021), neglecting the links between logic and language leads to the paradox that mathematical formalism, which should serve to clarify concepts, becomes an obstacle to students' learning. Indeed, logical formalism is often only seen when it is needed to express a mathematical concept not related to logic, and is viewed more as a syntactic abbreviation than a semantic clarifier. A student may encounter quantifiers for the first time in the limit formula—a formula featuring three quantifiers as well as an implication—simply because it is no longer possible to express the concept in words. Introducing formal symbols for quantifiers this late in the game feels like a missed opportunity, akin to introducing the equality symbol for the first time when dealing with equations.

Adopting a symbol to denote a concept requires a societal agreement on its meaning, and brings to the fact that symbols are related to the context of use (Ferrari, 2002). For instance, the logical conjunction AND will not capture every 'and' used in natural language; however, knowing how to recognise the differences and similarities in each case and context is an excellent starting point for learning the conjunction itself. Coppola *et al*. (2019) provide an interesting analysis of the relationship between language, as an object to manipulate and reflect upon, and the development of logical abilities, considering specific scenarios of social interaction among primary school children.

Our research question, which this work begins to explore, is whether logic supports the development of rational thinking and, consequently, whether it is appropriate to dedicate time and space to logic from the earliest stages of education onwards. A related research question is whether students—even as early as second grade—are capable of understanding and manipulating symbols not only at a syntactic but also at a semantic level. To address our research question, we set up and analysed a programme for introducing logic at primary school. We tested the programme in two primary schools: two second-grade classes of an Italian school attended by many foreign students, and a fourth-grade class of a French school. In our programme, every concept presented to the class is first introduced through discussions on language before moving onto the notation and formalism. As part of the programme, we introduce formal symbols to identify certain logical concepts, such as predicates and negation. The description of the trial run is based on field notes and recordings from both primary schools, and some of the key educational moments of the programme are explored. The educational programme was analysed from both qualitative and quantitative perspectives. The qualitative analysis was conducted through the lens of Duval's (2017) semio-cognitive theory. The quantitative analysis—supporting the qualitative one—aimed to investigate whether the explicit treatment of logic in primary education, in a game-like setting that integrates a variety of learning artefacts and dramatic approaches, improves students' logical-mathematical ability (mathematical literacy, as defined in OECD PISA, 2021) and cognitive abilities, in particular the so-called "fluid intelligence" (Kaplan & Saccuzzo, 2009).

Theoretical Framework

Our programme involves a playful approach to learning using a range of tools and techniques: theatrical activity, physical simulation of Boolean circuits to favour embodied cognition (Abrahamson & Bakker, 2016), discussions about symbols, worksheets on predicates, solving equations by trial and error, and an online game (Bernardi, 2022).

Two characters are present in all of these activities: the knight and the knave. The island of knights and knaves is a well known tale by Raymond Smullyan, describing an island whose inhabitants are either knights, who always tell the truth, or knaves, who always lie. This island is mostly known from logical puzzles in which one is tasked with identifying the inhabitants of the island on the basis of their statements, with such puzzles making an appearance as early as primary education (Carotenuto et al., 2017). Nonetheless, a story built around characters who speak the truth and characters who lie is likely to have a wider didactical value, not just because "putting these matters in human terms has an enormous psychological appeal" (Smullyan, 1987, introduction), but also because—owing to the use and acceptance of falsehoods (those uttered by the knaves)—it offers a more playful approach to errors, which become part of the learning process. If not properly addressed, false statements run the risk—in student's perception—of being seen as mistakes and thus as synonymous with failure and therefore to be avoided: as maintained by Zan and Di Martino (2017) this is usual in an approach commonly found in mathematical teaching that is focused on training—and thus focused on reproducible processes. However, avoidance of error limits the possibility of dialogic learning, based on the comparison between truth and falsehood.

Semiotic errors can be divided into three broad categories: structural errors (*i.e.*, syntactic), where symbols are used incorrectly (*e.g.*, a writing like 3 + + + = =), errors in meaning (*i.e.*, semantic), where symbols are used correctly but the meaning is incorrect (*e.g.*, $3 + 3 = 5$); and errors in interpretation (*i.e.*, pragmatic), where both syntactic and semantic aspects are fine, but the person writing or reading the statement does not interpret it as established by the community. A statement written with incorrect syntax does not have a truth value (it is neither true nor false), whereas a syntactically correct statement can be true or false. In our programme, the terms 'true' and 'false' are preferable to 'right' and 'wrong' when speaking about semantic errors: 'true' and 'false' have a clear logical meaning, and their use helps to place the two outcomes on equal footing. Furthermore, we prefer discussions on truth and falsehood to be done through role-play, where truth is represented by knights and falsehood is represented by knaves. Knights and knaves allow us to discuss semantic errors with ease, given that the errors made by knaves in their statements are always semantic, not syntactic nor pragmatic. A study by Rushton (2018)—who used incorrectly completed exercises for the purpose of student-conducted error analysis—showed an increase in mathematical understanding when working with a combination of correctly and erroneously completed exercises. Indeed, false statements can not only enhance understanding of symbols and concepts (*e.g.*, knowing that $3 < 2$ is false helps to clarify the meaning of the symbol $<$) but they can especially help bring to the surface the need for argumentation.

In the classroom, students are rarely allowed to roam freely through the world of mathematics. Such roaming entails trying and failing (the word 'error' is derived from the Latin word *errare*, to wander or stray), and then modifying their approach on the basis of the information acquired. We believe that the character of the knave—a character with whom students generally sympathise—can have a positive emotional return on mistakes and foster a process of inquiry through trial and error. When exploring primary school teachers' opinions on logic, Bibby (2002) found the majority believe that the objectivity of logic contrasts with the apparent subjectivity of the creative process, viewing logic as an obstacle to mathematical discovery. This belief seems to be based on a limited view of logic, in which logic is reduced to a syntactic formalism without semantic value and, above all, is considered a technique solely related to deduction, "assumed as an unproblematic foundation for the justification of knowledge" (Ernest, 1991, p. 6). Logic has a broader scope and can aid in the art of discovery.

Our module not only enables a different approach to error, but also permits an analysis of sentence structure. We argue that continually translating between the structure of a statement and its interpretation can favour “proceptual thinking” which, according to Gray and Tall (1994), is a key determinant of a “successful thinker” when it comes to development of cognitive and mathematical abilities. Indeed, the ambiguity of notation—*i.e*., the role of a symbol both as a process and as a concept—“allows the successful thinker the flexibility in thought to move between the process to carry out a mathematical task and the concept to be mentally manipulated as part of a wider mental schema” (ibid. p.115). Gray and Tall also believe that “mathematical symbolism is a major source of both success and distress in mathematics learning”, and that a successful thinker is able to “employ the simple device of using the same notation to represent both a process and the product of that process”. Our programme focuses on the study of semantics and its relationship with syntax. Through their island, the knights and knaves can help to define and delimit the context of the analysis and work to be done: the symbols we introduce can only be used on their island—in other words, within a logical and symbolic context—and not, for example, in an essay. First, we focus on the syntactic aspects of language and how they come together to create meaning. To do this, we look both at the words that make up our language and the structure that supports it—*i.e.*, the rules that allow us to move from words to syntactically correct sentences. Through symbolism, we can highlight the role of structure with respect to words.

We situate this discussion within the framework of Duval's semio-cognitive theory (Duval, 2017). Throughout our educational path, as we shall see, we consistently use what Duval terms *conversions*—transitions from one semiotic register to another—rather than *treatments*, transformations performed within the same semiotic register. Indeed, our programme encompasses aspects of logic and algebra but our students do not possess the tools necessary to perform transformations within symbolic-logical or algebraic environments. Therefore, every interpretation must involve a shift between registers, moving from the symbolic-logical to the linguistic register or from the algebraic to the arithmetic register. Conversions cannot be mechanically executed without comprehending the semantic relationship between the involved representations. To translate an expression into another register, students must recognise the logical-mathematical object it represents and understand how to express it within another semiotic system. Conversions are closely tied to the semantic dimension: one must interpret the initial representation to reformulate it correctly in a new register. It is therefore unsurprising that Duval identifies conversions as the primary source of difficulty in mathematical comprehension and emphasises the ability to coordinate multiple representational registers as fundamental for achieving a deep conceptual understanding of mathematical objects. Indeed, most researchers contend that the ability to translate between different modes of representation fosters a deeper understanding of mathematics (Lesh *et al.*, 1987).

Duval's idea of conversion is intrinsically linked to the notion of mental models, as constructing an appropriate mental model of a concept means semantically interpreting one semiotic register correctly. The importance of developing logically correct mental models at the primary-school level is highlighted by Khun et al. (2000). In their paper the authors argue that inquiry-based learning “at and from” middle-school level can be compromised by students having flawed mental models of causality. The paper shows how the simultaneous occurrence of a given value of a variable in a multivariable system and a particular outcome can be sufficient for students to expect a causal relationship between the variable and the outcome (in particular, the students struggle to conceive the outcome as being independent from the variable, and thus unaffected by the latter). The authors call this flawed model the co-occurrence model. This problem is not only due to a misunderstanding of causality, but also a failure to account for the additivity of the individual factors (*i.e.*, their combined contribution) within a multivariate system. We believe that a true understanding of causality and additivity can only be reached after previous study of logical connectives. On the one hand, logical implication—which has no causal value—illustrates how co-occurrence is not sufficient for causality; on the other, the use of the connectives *AND* and *OR* with independent variables helps to develop the mental model required to “deconstruct” the total effect into that of the individual factors, providing the background required to understand additivity. Although the activity proposed here does not cover logical implication and only briefly touches upon the connectives *AND* and *OR,* it nonetheless lays the necessary groundwork to address those topics. The study of logic in primary education—including the experiences proposed here—aims to prevent the development of the flawed mental models described in Kuhn, which impede examination of multivariate systems; in doing so, they prevent the possibility of carrying out inquiry-based learning (IBSE) in the higher educational phases, which is increasingly supported by the many educational institutions (as OECD PISA).

Description of the Trial Run

The activities presented here have been used in two second-grade classes (ages 7–8 years) of an Italian primary school and in a CM1 class (ages 9–10 years) of a French primary school, to introduce students to propositional logic (OILER, 2021). In the following, we refer to both the researcher and the class teachers who assisted as teachers.

Phase 1: Theatrical Activity.

The aim of the first activity was to introduce students to the knight and knave characters. Masks for the two character types were prepared in advance to be given to each student when needed. The knight character was introduced first, described as someone who always tells the truth: after being given an example by the teacher, the students were then encouraged to make true statements on any topic, while wearing a knight mask, taking turns to speak. Interestingly, the students immediately grasped the concept of a true statement: they began by providing statements about their immediate surroundings, and then moved onto general truths. Some statements were not verifiable (*e.g.*, “my sister is called Maria”), but they all had a truth value. No students suggested phrases that did not correspond to a statement, such as “the umbrella”, nor phrases without a truth value, such as “it will rain tomorrow”. The students were then encouraged to come up with mathematical statements once the teacher had provided an initial example of a simple identity such as “2 plus 2 is equal to 4”.

The knave character was then introduced as someone who always lies, and—as with the knight—the students were encouraged to make false statements, first generally and then within a mathematical context. Although the teacher's initial examples were of incorrect calculations, such as "2 plus 3 is equal to 2", some students opted for diverse examples of mathematical falsehoods, such as "100 has two figures".

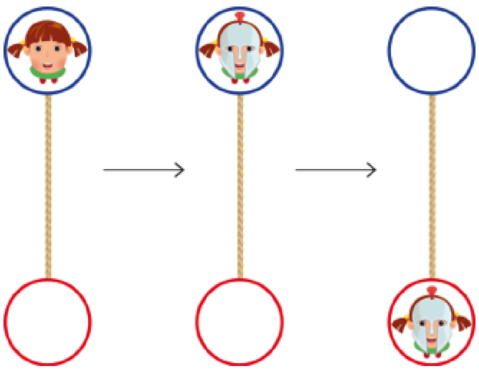
The next part of the activity was more similar to Smullyan's classic puzzles, where students had to work out whether the character speaking was a knight or a knave. The fundamental difference compared to Smullyan's classic riddles is that the puzzle is not presented in written form but through a theatrical activity: one cannot know whether the speaker is a knight or a knave simply because the person (who's wearing a mask) is facing away. This activity required two teachers, one who wore a knight or knave mask with their back turned to the class and provided riddles for the students, and another who helped the students to solve the riddles and identify which character was speaking. To start with, the riddles were very simple, as they did not follow the classic formulation seen in Smullyan's puzzles (which self-refer to the same group of characters speaking) but were simple statements such as "tigers can fly". The students were then asked in turn to play the role of the knight or knave and provide riddles for their classmates. The teacher then introduced the emblematic statement "I am a knight", always with their back turned to the class. After initial attempts to reach a decisive solution—during which both characters were suggested—the class realised that it was not possible to know whether the person speaking was a knight or a knave on the basis of that statement alone. This provides the students with an example of a question to which there is no single correct answer. Similarly, the class was encouraged to consider the phrase "I am a knave" and were pleased to discover that neither character would have been able to say this phrase. At the end of the first part of Phase 1, some of Smullyan's simpler classic riddles were proposed to the class, which involved more than one masked character with their backs to the class (one teacher and one or more students). The teacher told each of the masked students what to say and the rest of the class was asked to deduce their identity. It is important to note that creating physical representations of the characters making these statements—with their backs turned and faces hidden, but nonetheless there in person—is likely to have made it easier for the students to solve the riddles.

At the culminating part of the activity, the class were introduced to "Boolean circuits", using the knave as a representation of 'false' and the knight as a representation of 'true'. This choice works on a logical level, given that for every statement A made by a knave, we have $A \leftrightarrow$ FALSE, and for every statement B made by a knight, we have $B \leftrightarrow$ TRUE. A circuit is a path formed by ropes that begins on one or more blue circles, called *initial circles*, and ends on a red circle, called the *destination circle*. More formally, a circuit is a tree structure whose root is termed the destination circle, while the leaves are referred to as initial circles. Each circuit is simultaneously played by a team of students, with one student assigned to each initial circle. At the start, each student positions themselves on an initial circle and deliberately selects a mask to wear. The team's goal is to have one of their members reach the destination circle while wearing the knight mask. Throughout the path, students are not allowed to deliberately change masks; however, external events may occur that either alter masks or remove certain masks from play.

The first proposed circuit is trivial, featuring only one initial circle (thus involving only a single student). The student merely has to follow the rope to the destination circle without encountering any unexpected events along the way (Fig. 1). The winning strategy, therefore, consists of choosing the knight mask at the initial circle and proceeding directly to the destination circle. If the knave mask is chosen, the student loses, as they would reach the destination circle wearing the incorrect mask. To make the game faster, multiple circuits were created so that the class could play simultaneously in several distinct groups.

Figure 1

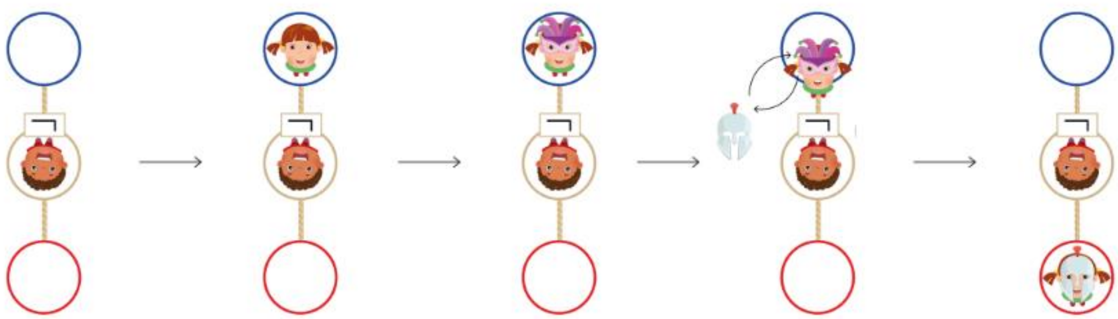
The First Circuit Proposed to the Classroom



Note: The student enters the initial circle (step 1), chooses to wear the knight mask (step 2), and walks along the rope, reaching the destination circle while still wearing the knight mask, thus winning (step 3).

Figure 2

The Second Circuit Proposed to the Classroom



Note: Dr. No is positioned midway along the path (step 1). The student enters the initial circle (step 2) and chooses to wear the knave mask (step 3). The student walks on the rope and encounters Dr. No, who forces the student to change their mask (step 4). The student walks along the rope, reaching the destination circle while wearing the knight mask, thus winning (step 5).

The second circuit presented to the class introduced Dr. No, a character (played by a student) who forces any player encountering them to change their mask. Dr. No wears, as their distinctive symbol, the formal symbol for negation (*i.e.* ¬). The second circuit resembles the first, with the only difference being the appearance of Dr. No along the path. In this scenario, the winning strategy is to begin wearing the knave mask, as encountering Dr. No will cause the mask to change, allowing the student to reach the destination circle wearing the knight mask (Fig. 2).
The next circuit then included two Dr. No's (Fig. 3), one after the other; here, the winning strategy is to start the circuit wearing the knight mask, because the mask is changed twice. More Dr. No's were then introduced sequentially into the circuits, leading towards a discussion on the parity of the number of negations: if there is an even number of Dr. No's—including none at all—the winning strategy is to start with the knight mask; if there is an odd number of Dr. No's, the winning strategy is to start with the knave mask. After a few initial mistakes, all of the students understood the winning strategy and were able to choose the mask needed to successfully complete the circuit. The relationship between the winning strategy and the parity of the number of negations was highlighted.

Figure 3

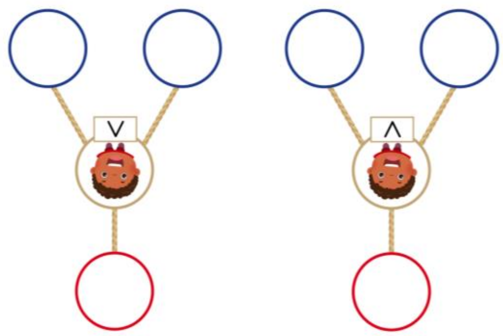
Multiple Negations Circuit

Note: Depending on the number of Dr. No characters present in the circuit, the winning strategy is either to start with the knight mask or with the knave mask.

The two final circuits introduced to the class represented more complex cases, where the student team no longer consisted of just a single student. Additionally, two new characters, Dr. OR and Dr. AND, were introduced. Dr. AND is a character who prefers knaves: if approached by a knight and a knave, Dr. AND will let the knave pass; if approached by two knaves, they will let the knave of their choosing pass; and if approached by two knights, they will be forced to let a knight pass. Dr. OR is a similar but opposite character to Dr. AND, instead preferring knights: if approached by a knight and a knave, Dr. OR will let the knight pass; if approached by two knights, they will let the knight of their choosing pass; and if approached by two knaves, they will be forced to let one of the knaves pass. The two proposed circuits are depicted in Fig. 4. The class successfully completed the circuits, observing that in the case involving Dr. OR, multiple winning strategies existed (it is sufficient for at least one of the two students to be a knight to win). As we can see, this activity has a significant embodied component in which the *treatment rules* (Duval, 2017) for negation, AND, and OR within the circuits are experienced first-hand.

Figure 4

Circuits with AND and OR



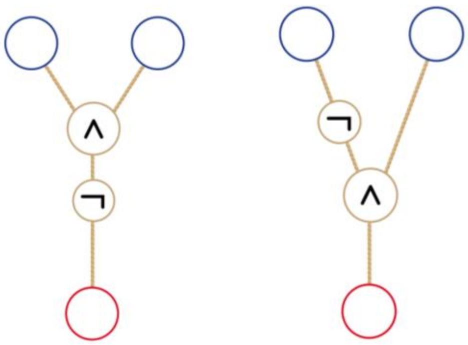
Note: This way of using AND and OR—based on the preferences of Dr. OR and Dr. AND—corresponds exactly to the truth tables of the two connectives, positioning each connective as a rule of deduction rather than a symbol with a particular meaning.

The activity with the fourth-grade class followed the same steps as with the second-grade class, but at a quicker pace. Notably, a student made a statement that triggered the discussion of a key point, declaring “we are all girls” while wearing a knave mask; this statement opened the floor to a discussion on the terms ‘all’, ‘at most’, ‘at least’, and ‘none’. When initially questioned, the students fell foul of the classic mistake of thinking the negation of ‘all’ to be ‘none’, and vice versa. To address this, the teacher laid a pink hula hoop on the floor and said “all of the circles are pink”; the class correctly identified the teacher as a knight. The teacher then put another pink hoop on the floor and repeated the phrase. Again, the class correctly identified them as a knight. The teacher added yet another pink hoop to the collection, and the discussion was repeated. Finally, the teacher laid down a green hoop: at this point, there were three pink hoops and one green hoop on the floor. The teacher then said “all of the circles are pink”, at which point the class correctly identified them as being a knave. In this way, the class was able to explore the key concept by which the negation of ‘all’ is ‘there exists one that is not’. Knights and knaves are a useful tool for the analysis of such words: negation of quantifiers can be hard to grasp, but identifying with a character who lies or speaks the truth can aid in understanding. With regard to circuits, in addition to presenting the same circuits introduced in the second grade, students were invited to design their own more complex circuits to be submitted to their peers. Two examples of such student-designed circuits are shown in the Fig. 5.

To consolidate the concepts learned in this phase, and to approach them from another perspective, the classes played Bul Game before moving onto Phase 2. Bul Game is an online game in which players encounter either a knight or a knave in each turn, and progress through the game by choosing between two options on the basis of a statement given by the knight or knave (Bernardi, 2022). The types of question encountered are classified into sections that reflect the stages of the activities.

Figure 5

Circuits Proposed by the Students



Note: The students also proposed more complex circuits, enjoying the challenge of incorporating numerous negations.

Phase 2: Predicates

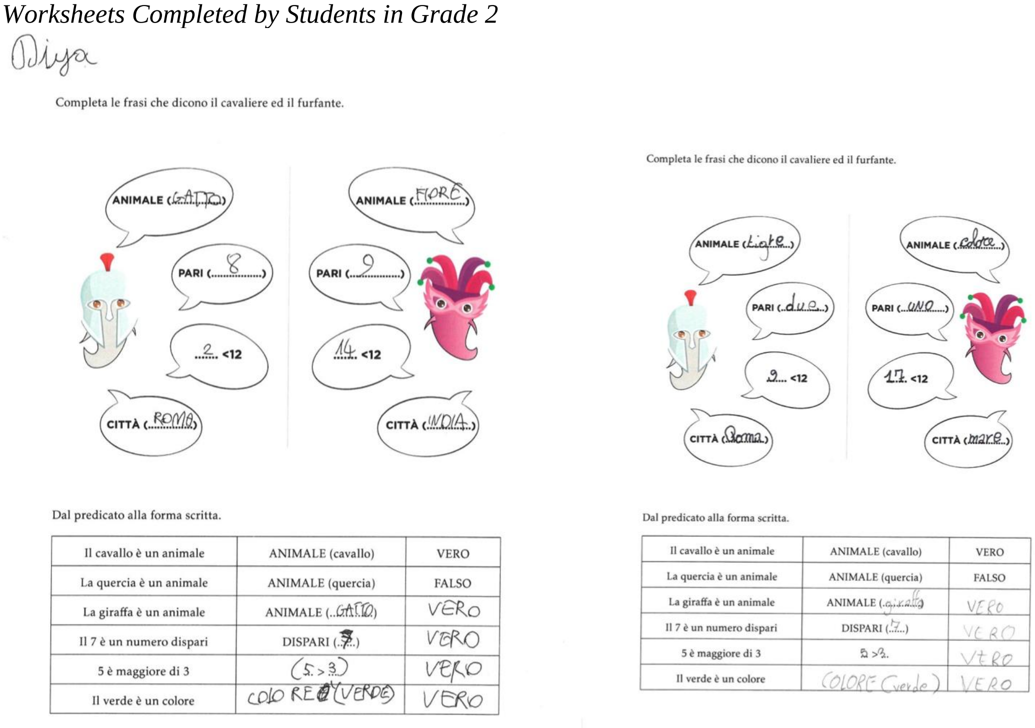
The students were told that knights and knaves sometimes communicate with one another using a strange way of writing. First of all, students were asked to pick out the key elements of a phrase such as “a tiger is an animal”, identifying ‘tiger’ and ‘animal’ as essential words to understand its meaning. More accurately, the central components of the phrase are the predicate “being an animal” and the object (in this case, the subject of the phrase) that the predicate refers to. Anticipating the next step, a student noted that “a ‘not’ would be important too if it were there”. The students were then given parentheses to colour in, to familiarise them with the symbol. The students were then told that knights and knaves use the two words ‘tiger’ and ‘animal’ and parentheses to write the phrase “a tiger is an animal”. Some students suggested TIGER(ANIMAL) as a potential solution, and others (TIGER ANIMAL) (which is somewhat reminiscent of Barendregt’s lambda calculus!); a few other students suggested ANIMAL(TIGER). Each of these three notations can be used without leading to contradictions. The students were finally told that knights and knaves use the notation ANIMAL(tiger). This is the standard notation used in logic and general mathematics, where the object of the predicate, or function, sits within parentheses after the symbol for the function.

We feel that this early introduction of formal notation can be beneficial: first, as mentioned in the introduction, it allows students to become accustomed to using a symbolic and context-dependent language (this language is used exclusively on Smullyan’s island of the knights and knaves). This situation highlights the fact that changing language does not necessarily involve changing the vocabulary or alphabet; the formal language outlined here shares the same words and symbols as English, but applies them using different rules. The key point is to create a broader view of language, which is not defined exclusively by its alphabet and vocabulary but also by the rules that govern the construction of phrases (Bernardi, 2022). Furthermore, by considering a range of objects that either verify or falsify a given predicate, one is gradually able to identify and isolate the specific characteristics—*i.e.*, the properties—that characterise objects that satisfy that predicate. In other words, notation such as ANIMAL( ) encourages the transition from an extensive description (based on many examples) to an intensive description of being an animal. Finally, the formal structure of predicates makes it easier to write phrases with the negation symbol, as we will see in the following phases.

Importance was placed on the translation from symbolic form to natural language: TREE(oak) should not be read “tree oak” but “an oak is a tree”. The students were given statements to translate in both directions, with examples of true statements—*i.e.*, those made by a knight—such as ANIMAL(tiger), and false statements—*i.e.*, those made by a knave—such as ANIMAL(table). To finish the activity, students were given worksheets in which they were asked to correctly complete predicates according to which character was speaking—for example, EVEN(…) or … < 12—and to translate from natural language to logical language (Fig. 6).

Figure 6

Worksheets Completed by Students in Grade 2



Note: To interpret the figure, note that "pari" means even, "dispari" means odd, "animale" means animal, “gatto” cat, “fiore” flower, “cavallo” horse, “quercia” oak, “verde” green, “mare” sea, "vero" true, and "falso" false. For instance, “Il cavallo è un animale” means: the horse is an animal, *i.e*. ANIMAL (horse).

It should be noted that the students were free to fill in the predicates as they pleased. If a knight is speaking and we write CITY(x), x is necessarily a city. But if a knave is speaking, x can be anything that is not a city. Nonetheless, most students favoured the more meaningful contexts: NOT CITY(France) makes more sense than NOT CITY(9).

Phase 3: Negation

The phrase “a tiger is not an animal” was written on the board and the students were asked, as in Phase 2, to identify the key words. It was noted that, in addition to ‘tiger’ and ‘animal’, the word ‘not’ was also fundamental. A few exercises were done on the board whereby students needed to work out whether a given phrase had been said by a knight or a knave; for example, the first phrase was said by a knave, whereas the phrase “3 is not even” was said by a knight. The class was then given the negation symbol ¬ to colour in, to familiarise them with the symbol. Many recognised the symbol from the first phase, when it was used with the Dr. No character. Following this, the students carried out translation exercises—first orally at the board, and then written—and were given comics to fill in, depending on whether the person speaking in the comic was a knight or a knave. The phrase “red is not a colour” would be translated as ¬ COLOUR(red). Similarly, the phrase ¬ ODD(4) is translated as “4 is not an odd number”.

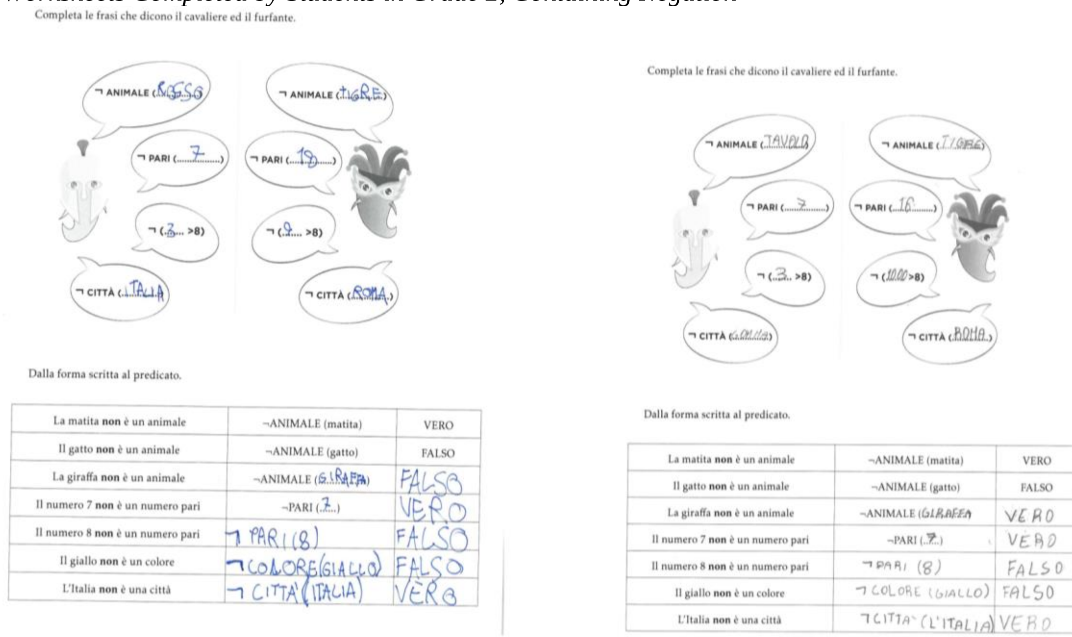
We highlight here that two different approaches were taken for the negation symbol. In Phase 1, the symbol was introduced as a rule: the symbol acted on the truth value of a statement by changing it—that is, by changing the mask worn. In Phase 3, the negation symbol was introduced as a logical connective with semantic value. These two interpretations are clearly very closely connected. If either a knight or a knave writes the phrase PREDICATE(object), then the introduction of the negation symbol will force a character swap, because the phrase ¬PREDICATE(object) can only be written by the other character.

Until this point, the statements provided that contained the negation symbol had been limited to the form ¬PREDICATE(object). To reinforce students’ understanding of negation, a series of worksheets has been proposed (see Fig. 7). In the first part of the worksheet, students are tasked with completing statements made by knights and knaves in the presence of negation. It is interesting to note that now, due to the presence of negation, students will complete a knight predicates so that it becomes false, and a knave predicate so that it becomes true. In this way, a different perspective is offered on the aspect previously seen in the circuits where Dr. No made the mask change. While the circuits focused more on the operational and procedural aspects of negation, here the emphasis is placed on its semantic dimension.

It is worth noting that in natural language, the position of a negation is often not well defined a priori; in some cases, moving its position does not affect the meaning of the phrase (*e.g.*, “all people do not have blonde hair” is equivalent to saying “all people have non-blonde hair”), whereas in other cases, moving the negation can distort the meaning (*e.g.*, “not all people have blonde hair” is completely different from “all people do not have blonde hair”). Further examples relate to double negations, which work as affirmations in some languages and negations in others. We believe that introducing the negation symbol and its rules aids in the understanding and clarification of these various situations.

Figure 7

Worksheets Completed by Students in Grade 2, Containing Negation



Note: As an example, "la matita non è un animale" means: the pencil is not an animal, *i.e.* ¬ANIMAL (pencil). Moreover, note that "rosso" means red, “giallo” yellow, "tavolo" table.

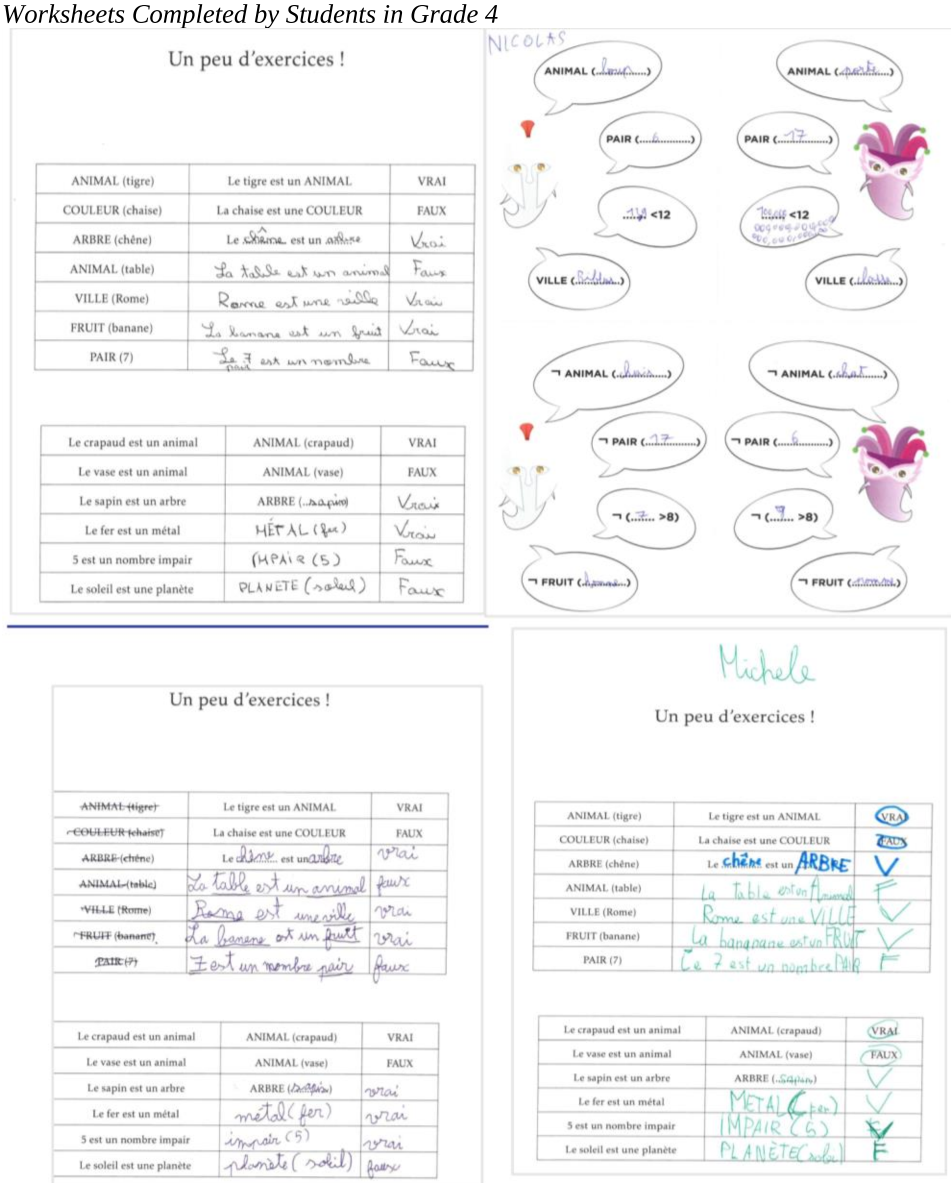
For the fourth-grade class, phases 2 and 3 of the activities were merged, making the previously described dynamic of negation that "changes the mask" even more explicit (Fig. 8, top right). Furthermore, binary predicates involving two objects were introduced. The first binary predicate to be introduced was MOTHER(x, y), with the teacher going through several examples with the students; the chosen convention was that x is the mother of y. Examples of this predicate were given where a knight was speaking, as well as where a knave was speaking. The predicate FRIENDS(x, y) was then introduced, with further examples. It was noted that writing MOTHER(x, y) is different to writing MOTHER(y, x) (in fact, one case precludes the other), whereas writing FRIENDS(x, y) is equivalent to writing FRIENDS(y, x). This property of the latter was described to the class as symmetry, which is common in mathematics: for example, the binary predicate “<” is not symmetrical whereas the binary predicate “=” is.

Phase 4: Variables (Gym Hall)

This part of the programme concerns the search for a solution of an equation via trial and error. We note that a single-variable equation is a particular type of unary predicate. The main activity involved laying out numerous cards, some containing equations, inequalities, or predicates on the floor and other cards with numbers (Fig. 9). The aim was to complete the equations, inequalities, or predicates—such as EVEN(x)—by placing an appropriate number over the x; in other words, substituting a constant for a variable.

Figure 8

Worksheets Completed by Students in Grade 4



Note: To interpret the figure, note that “chaise” means chair , “arbre” means tree, “chêne” oak, “crapaud” toad, “sapin” fir tree, “fer” iron, “impair” odd, “pair” even, ”soleil” sun, “chat” cat, “vrai” true.

Such equations, which are generally first encountered in middle school (ages 11–14 years), are usually solved through *treatment* of algebraic symbolism (Duval, 2017). We clearly did not consider it appropriate to introduce such a method at primary school and the equations were instead solved by trial and error: different numbers were substituted for x and the resulting equality was checked. If the equation was correct, it was considered solved;

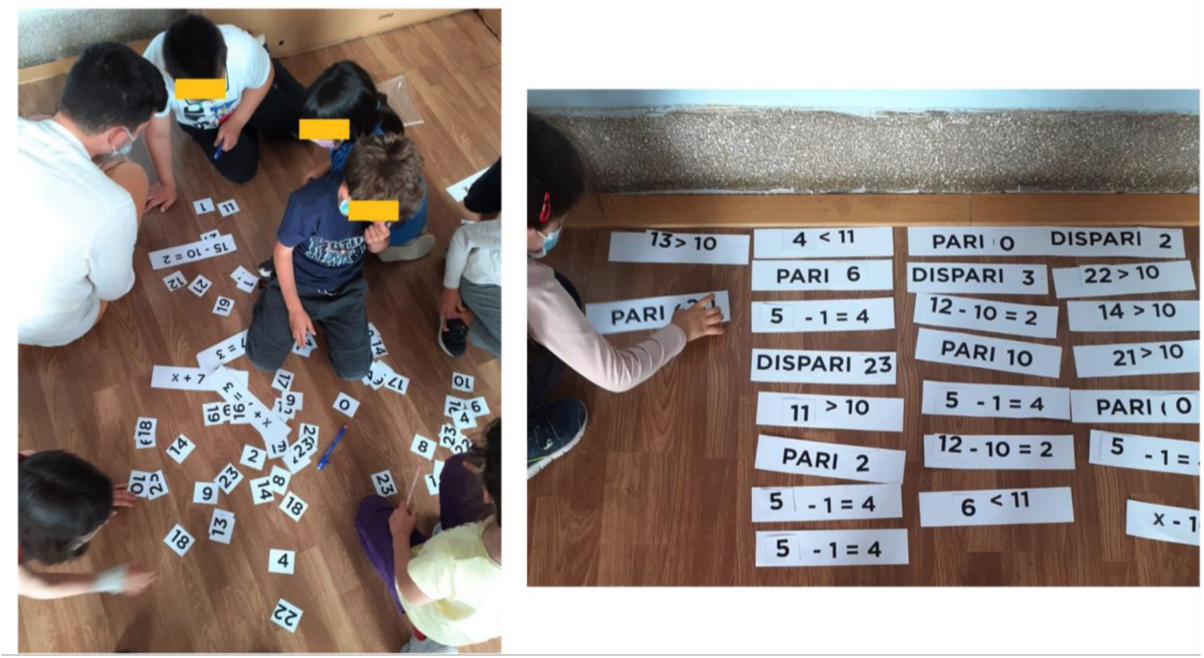
otherwise, a different number was tried. In other words, since the students did not know how to perform algebraic treatments, they were forced to convert everything into an arithmetic register—a register in which they are able to carry out treatments and verify equalities. The use of the knave character appears to support children during the trial-and-error processes by alleviating the fear of making mistakes.

More complex statements were then proposed to the class, such as EVEN(*x*), and the class noted that, this time, there were many different possible solutions. Multiple requirements were therefore added together: "I know a number x such that EVEN(*x*), *x* <10, and *x* is a three-letter word. What number is it?" In this case, there is still more than one solution, but the number of solutions is finite. The children were allowed to work freely, and enjoyed coming up with numbers they wanted to substitute for x, trying out a wide range of numbers. Once an equation was solved by a student, it was put aside (Fig. 9 right). By that time, the class was used to recognising false statements (and judging them as such) thanks to their familiarity with the knave character. We believe that solving through trial and error should also be encouraged after primary school to make students more comfortable with the meaning of the equation itself.

In grade 4, the terminology already discussed in Phase 1, such as ‘all’, ‘at least one’, and ‘none’, was reviewed through examples, as was the concept that the negation of ‘all’ is ‘at least one is not’ and the negation of ‘none’ is ‘at least one is’—all with the help of knights and knaves. The concept of a variable was then introduced, using similar examples to those used in the second-grade class. This concept was then incorporated into the previous activity. A formula was written on the board that contained x and the students were asked whether the elements x that satisfied the formula (referring implicitly to natural numbers) were all, some (*i.e.*, at least one; the students did not seem to have a problem with it being exactly one), or none. For example, x + 3 = 5 is satisfied by one number, whereas x = x is satisfied by all numbers; by contrast, x > x and x + 3 = 1 are not satisfied by any natural number. The same question was posed about the predicates EVEN(x) and ODD(x), noticing that, even if not all numbers satisfied the predicates, both were satisfied by infinitely many numbers. Furthermore, it was pointed out that EVEN(x + x) is satisfied by all natural numbers. The class was asked to find an equivalent expression such that ODD(expression) was true for all natural numbers. At first, the class had no idea how to approach this problem, but then began to work out what sort of expression would be required. They were placed into small groups to work on a solution, supported by three teachers. The students suggested solutions such as ODD(x – x + 1): they were told that, while correct, these expressions always give the same result, regardless of the value of x, and were encouraged to find a non-constant expression. After a while, several students independently concluded that a possible solution was ODD(x + x + 1). The rest of the activity followed the same steps as for the second-grade class, with numbers laid out on the floor, alongside formulas to be completed, with a few more complex examples introduced.

Figure 9

Playing with Equations and Putting Predicates Aside



Note: The teacher and the students are observing the writing "15 – 10 = 2" wondering if it is right or not. To interpret the figure, note that "pari" means even, "dispari" means odd.

Discussion

Qualitative Observations

The educational sequence was implemented in three classes, each of which received five lessons lasting approximately two hours. One of the research questions concerned the appropriateness of the sequence for younger students, *i.e.*, those in second grade, and whether any aspects might be considered too premature. To gain insights into this question, we relied on field notes of the researchers involved in the study and on the worksheets filled in by the students. The observations tell us that students participated actively and that certain objectives can be considered acquired, such as—for example—to distinguish and construct true and false statements; distinguish subject and predicate in a sentence; use formal notations for predicates and negation; construct, in the context of predicates, true statements and false statements; recognise the crucial role that negation plays in the meaning of a sentence; translate non-trivial symbolic writings into current language and vice versa; and solve simple equations by trial and error.

In general, the experience proved to be meaningful for the students from the very first activity. They were highly engaged and enthusiastic about taking on the role of the knave, and the act of telling lies was not merely a passive replication of the teacher's example; rather, the students made an effort to be creative in formulating their own statements. Even in the mathematical domain, although the teacher had simply suggested using an incorrect calculation, many students went further, offering different observations that they considered meaningful—such as

“100 has two figures” or “a triangle has four sides”. In the first phase of our sequence, symbols are also examined through an operational and algorithmic register (within the context of logic circuits). In this way, students explore conversions between the algorithmic register (deduction rules), the symbolic register, and the linguistic register.

As far as predicates and negation are concerned, it is interesting to note, within Duval’s theoretical framework, how the logical register compels a conversion into a linguistic register, since the students lack the knowledge needed to work with syntactic representations. For example, after introducing the negation symbol (simply by saying that knights and knaves on their island use the symbol “$\neg$” to express negation), the statement $\neg(3<2)$ was written on the board. A student was asked to translate it. Notably, the student translated it as “3 is not less than 2”, applying the negation to the predicate, showing a correct understanding of symbolic language as well as a solid ability to convert between registers. The worksheets related to negation confirm that, by and large, both second-grade classes adequately understood the symbol and the underlying concept.

Negation is a concept of fundamental importance in both logic and mathematics. For other concepts likewise considered essential—such as equality and addition—the necessity of symbolic representations from the first years of primary education has always been universally recognised (Kieran, 1981). Just as the equality symbol supports the development of language and relational thinking, we consider the negation symbol to play an analogous role in the development of language and rational thinking.

Let us return to the students’ productions shown in Fig. 6 and Fig. 7. The tendency to insert objects relevant to the predicate, regardless of whether the statement is false, was observed in a lot of cases, in particular with reference to predicates concerning mathematics. This is an interesting aspect, as it highlights not only that the activity was understood, but also that there is a strong semantic interpretation of what is being done. For instance, the sentence "3 is even" is perceived as more meaningful than "Paris is even", with a relatively well-defined—though not explicit—domain of objects. Comparing the two worksheets in Fig. 6, we notice that the first student writes ANIMALE(fiore)—*i.e.*, ANIMAL(flower)—and CITTA(India)—*i.e.*, CITY(India)—when the speaker is the knave, while the second writes ANIMALE(colore)—ANIMAL(colour)—and CITTA(mare)—CITY(sea). The first student's control over the domain is certainly stronger, and this control is likely linked to a greater ability to perform conversions between different registers.

The activity on equations offers the perfect opportunity to discuss proceptual thinking, as outlined in the theoretical framework. Children of primary-school age are not yet able to manipulate equations (*i.e.*, moving elements from one side to the other) and thus cannot carry out the “process” represented by the equation: an equation—before any substitutions—is simply a concept. Once a constant (number) has been substituted into the equation, the equation is processed. If the equality obtained is false, you go back to the original concept, more knowledgeable than before. For both Duval and Tall, the ability to convert meaning across multiple registers of representation is essential for a deep conceptual understanding of mathematical objects. As we can see, the central focus of the entire educational path lies in this act of conversion, since students are presented with environments in which they lack the ability to perform treatments. The only way for them to navigate such situations is to interpret the meaning and shift to a different register—moving, for instance, from symbolic logic to natural language, or from algebraic to arithmetic representation.

An Empirical Exercise

To complete the qualitative observations we conducted a *quantitative* analysis which allows us to deepen our analysis, even if limited to one class. The quantitative analysis aims to assess the potential impact of the intervention on students' mathematical literacy and cognitive abilities.

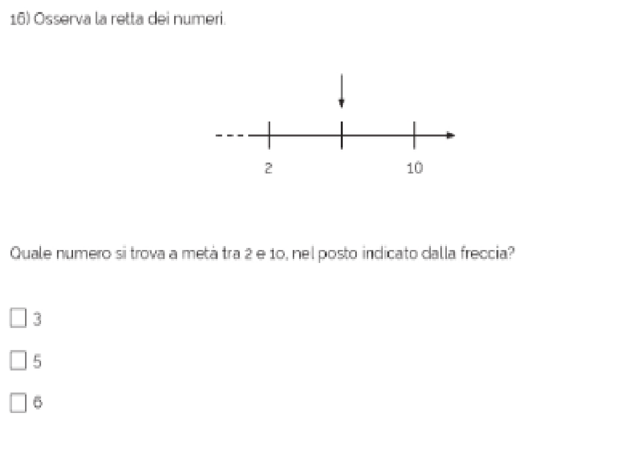
Methodology

Only one of the two second-grade classes in which the intervention was implemented was selected for qualitative evaluation. This choice was due to the fact that the teacher of that class also taught an additional class, which was then selected as the control group. The teacher was informed about the activity and participated in the sessions as an observer but was not made aware of the study's objectives—namely, what was being tested and how.
Therefore, our quantitative analysis focuses on one of the two second-grade classes where the intervention took place (hereafter referred to as the *intervention class*) and an additional control class in which the intervention was not conducted. The intervention class consisted of 21 students, while the control class comprised 20 students.
The intervention, as previously described, was carried out over the course of two months and included five sessions, each lasting two hours. A pre-test and a post-test were administered, respectively, before and after the intervention. To avoid potential sources of endogeneity, the control class was selected from the same grade level as the intervention class.

We validated the RCT (Randomised Controlled Trials) assumption of randomisation by using a t-test to verify that class compositions were effectively random with respect to relevant covariates (age, sex, and nationality of origin). In other words, the t-test is used to verify whether the two groups are similar in terms of observed characteristics. We measured mathematical literacy and cognitive abilities in both classes, before and after the intervention, and calculated the score difference. We measured mathematical literacy using INVALSI questions. INVALSI are national tests specially designated and recognised by the Italian state to evaluate skills (understood as knowledge and ability to think about knowledge) in fundamental areas such as mathematics, Italian, and English. Questions from the mathematics INVALSI tests therefore provide a good measure of mathematical literacy. The pre-intervention and post-intervention tests used in this trial were composed of different sets of four past INVALSI questions. An example of one of the INVALSI questions used is shown below (Fig. 10):

Figure 10

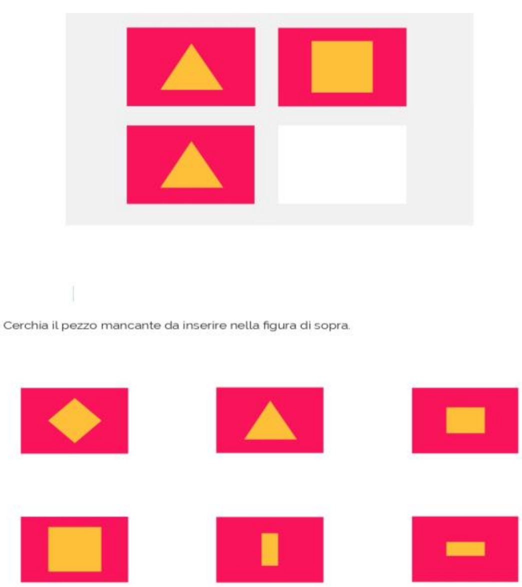
In the Question Shown Below the Student is Asked to Say which Number Lies Between 2 and 10



We measured cognitive abilities with Raven's progressive matrices. This non-verbal test is widely recognised as a measure of fluid intelligence, which refers to the ability to solve novel reasoning problems, and is correlated with several important skills such as comprehension, problem solving, and learning in individuals aged 5 years and older (Kaplan & Saccuzzo, 2009). We used 13 questions from the Coloured Progressive Matrices (RCPM) variant, a version of the Raven test designed specifically for children aged 5–11 years (Domino & Domino, 2006). An example question is shown below (Fig. 11):

Figure 11

A Question From the RCPM Matrices, where the Student is Asked to Choose the Missing Piece



We considered this type of test to be a reliable proxy of general cognitive skills, as a review of the psychological literature suggested that no factor of intelligence is independent from the g factor, or general intelligence factor, a construct developed to identify the common core of all cognitive tasks (Jensen, 1978). It is important to note that the RCPM is a non-verbal test of cognitive skills, while the educational programme being evaluated is closely linked to verbal reasoning, much like logic itself. The intervention therefore does not train students to complete the RCPM, and thus any changes in RCPM score after the intervention will represent a genuine change in cognitive skills. All the questions were equally weighted in both tests.

Data

From the original dataset, we excluded those who had not completed both the pre- and post-test, as well as those who scored the maximum on the first test—and therefore could not show improvement. Our final dataset was made up of 18 students in either class (Control e Intervention), whose characteristics are summarised in Table 1.

Table 1

Characteristics of the Students; RCPM Score Measures Cognitive Abilities

	Mean RCPM score	Males (%)	Non-Italian origin (%)
Control (N=18)	10.3	9 (50%)	5 (28%)
Intervention (N=18)	10.2	11 (61%)	5 (28%)

Here, we have included all observable variables that may affect the rate of improvement in the tests used. As the students were all of the same age, we performed a balancing test (Student’s t-test) for sex, nationality of origin, and initial RCPM score. Given the relationship between learning and intelligence outlined by Jensen (2006), it is possible that initial cognitive ability can affect the rate of learning of both mathematical literacy and cognitive skills. Furthermore, according to Vaci et al. (2019), the benefits of practice increase with intelligence, suggesting that a child with higher initial cognitive skills would be able to improve their mathematical literacy more than their peers just from the standard math classes. We therefore included the initial RCPM score in the relevant characteristics.

As already mentioned, to assess the randomness of class compositions, we used a Student’s t-test to compare the mean values of each covariate between the two groups; we chose this statistic because the variance of each covariate was similar between groups, and Student’s t-test is quite appropriate for very small samples (de Winter, 2013). As highlighted in De Winter's study, the t-test has a power of 80%, meaning that in the remaining 20% of cases, it fails to detect differences when they exist. This implies that the test is not entirely reliable for small groups, but it still provides a solid foundation for analyses—like ours—that do not aim to be definitive. The results are shown in Table 2:

Table 2

Results of the Student's T-Test Conducted Between the Intervention Class and the Control Class

	RCPM score	Sex	Non-Italian origin
t statistic	–0.23	1.07	0
p-value	0.42	0.62	1

The results show no significant difference between the two groups, validating the assumption of the class compositions being as good as random. Indeed, p value shows that observed differences between means in each covariate are not statistically significant. For both mathematical literacy and cognitive abilities, we ran two unadjusted regressions with score difference as the independent variable (Δy_i) and belonging to intervention group as the dependent dummy variable (x_i). In this analysis, the regression coefficient of the intervention variable represents the mean difference in the outcome between the intervention and control groups. Since the sample was relatively small, we set the significance level at $p = 0.1$.

Specifically, the regression equation is the following $\Delta y_i = a_0 + a_1 x_i + a_2\, sex_i + a_3 origin_i$. Recall that we used two regression equations, one relating to mathematical literacy and one relating to cognitive abilities. The variable y_i is the outcome variable related to the student i, Δy_i represents the difference between pre- and post-intervention scores. The coefficients $a_0, a_1 \ldots$ are the regression parameters and indicate the effect of each corresponding variable; x_i is a dummy variable (0 for the control group, 1 for the treatment group). Our main interest lies in the coefficient of the dummy variable: if it is positive and statistically significant, it supports the effectiveness of the intervention. In other words, this coefficient answers the question: *what is the effect of belonging to one group or the other?* An important issue that we were unable to adjust for is the potential influence of a memory effect. As the post-intervention RCPM test was composed of the same questions as the pre-intervention test, our results may be biased by this effect, even if the students were not given solutions to the tests. Anyway, memory effect is likely to affect both groups same way.

Results

We observed a marginally significant effect ($p < 0.1$), of the intervention on both cognitive skill and mathematical literacy. Notably, the regression coefficient of the dummy variable for cognitive skills is smaller than that for mathematical literacy (see Table 3), indicating a smaller difference in scores between the two groups, but the smaller p value indicates it approached acceptable levels of statistical significance ($p = 0.05$).

Table 3

Results of the Experiment

	Cognitive abilities	Mathematical literacy
Coefficient	0.55	2.27
Standard error	0.29	0.88
p-value	0.06	0.09

The RCT performed show that the educational programme proposed helps to develop mathematical literacy and general cognitive skills, specifically fluid intelligence, stimulating the formation of mental models that support the continued development of skills and abilities throughout the various stages of education. As argued in the theoretical framework, the improvement that takes place stems from a variety of sources: development of thought at the metalevel via in-depth study of the relationship between syntax and semantics, analysis of error, familiarity with logical symbolism, and stimulation of proceptual thinking.

Conclusion

The results presented in the quantitative experiment cannot provide a conclusive answer to our hypothesis, but they allow, on one hand, to support the qualitative analysis, and on the other hand, to provide an example of how to work with small size samples in the field of mathematics education. Our theories and empirical findings support the existing literature (Durand-Guerrier, 2021; Ferrari & Gerla 2015; Coppola et al., 2021), all of whom advocate for the introduction of the study of logic from primary education onwards, and argue that the study of language can help to develop logical abilities. Our next step will be to further explore the relationship between logic education and Duval's semio-cognitive theory, and to conduct a more detailed analysis of students' emotional responses to these programmes—for example, their engagement with and perception of mathematics.

Acknowledgements

The authors sincerely thank Giorgia Damiano, a teacher without whom the programme would not have been possible; Alessandra Palegi, Concetta Guerrieri, and Corine Pozzi teachers of the three classes; Emmanuel Beffara for helping in the development of the programme; Lorenzo Tortora de Falco and Paolo di Giamberardino for the work carried out at the French school. The authors particularly thank Giulia Vivaldi for her editing and suggestions. The authors sincerely thank the reviewers for their valuable and thorough work.

References

Abrahamson, D., Bakker, A. (2016). Making sense of movement in embodied design for mathematics learning. *Cogn. Research 1*, 33. http://doi.org/10.1186/s41235-016-0034-3

Bernardi, L. (2022). Bul Game: Playing With Knights and Knaves. In C. A. Huertas-Abril, E. Fernández-Ahumada, & N. Adamuz-Povedano (Eds.), *Handbook of Research on International Approaches and Practices for*

Gamifying Mathematics (pp. 170–188). IGI Global. http://doi.org/10.4018/978-1-7998-9660-9.ch009

Bibby, T. (2002). Creativity and Logic in Primary-School Mathematics: A View from the Classroom. *For the Learning of Mathematics*, *22*(3), 10–13. https://www.jstor.org/stable/40248395

Carotenuto, G., Coppola, C., & Tortora, R. (2017). Educating to rationality in a narrative context: An experimentation. In T. Dooley & G. Gueudet (Eds). *Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education* (CERME, February 1-5, 2017), (pp. 1252-1259), DCU Institute of Education & ERME.

Coppola, C., Mollo, M., & Pacelli, T. (2019). —The worlds' game: collective language manipulation as a space to develop logical abilities in a primary school classroom. *European Journal of Psychology of Education, 34*(4), 783–799. http://doi.org/10.1007/s10212-018-0401-1

Domino G., & Domino, M.L. (2006). *Psychological testing: An introduction*. Cambridge University Press. https://doi.org/10.1017/CBO9780511813757

Durand-Guerrier, V. (2021). The place of logic in school curricula and advocacy for logic at school. *Virtual conference, March 4-7, 2021: Panel in Logic Education*, ICLA 2021.

Durand-Guerrier, V., Boero, P., Douek, N., Epp, S. & Tanguay, D. (2012). Examining the Role of Logic in Teaching Proof. In G. Hanna et M. de Villier (Eds), *ICMI Study 19 Book: Proof and Proving in Mathematics Education*, chap. 16, (pp. 369-389). Springer. https://doi.org/10.1007/978-94-007-2129-6_1

Duval, R. (2017). *Understanding the Mathematical Way of Thinking – The Registers of Semiotic Representations*. Springer. http://doi.org/10.1007/978-3-319-56910-9

Ernest, P. (1991). *The Philosophy of Mathematics Education*. Routledge. https://doi.org/10.4324/9780203497012

Ferrari, P.L. (2002). Developing language through communication and conversion of semiotic systems. In A.D. Cockburn & E. Nardi (Eds.), *Proc. of the 26th Conf. of the Intern. Group for the Psychology of Math. Ed*. Norwich (UK), 2, (pp. 353–360). University of East Anglia.

Ferrari, P. L., & Gerla, G. (2015). Logica e didattica della matematica. In H. Hosni, G. Lolli, & C. Toffalori (Eds.), *Le direzioni della ricerca logica in Italia* (pp. 143–184). Pisa: Edizioni della Normale.

Gray, E., & Tall, D. (1994). Duality, ambiguity and flexibility: A proceptual view of Simple arithmetic. *Journal for Research in Mathematics Education*, *25*, 115-141. http://doi.org/10.2307/749505

Kaplan, R., & Saccuzzo, D. (2009). *Psychological Testing: Principles, Applications, and Issues*. Wadsworth.

Kieran, C. (1981). Concepts associated with the equality symbol. *Educational Studies in Mathematics*, 12(3), 317–326.

Kuhn, D., Black, J., Keselman, A., & Kaplan, D. (2000). The Development of Cognitive Skills to Support Inquiry Learning. *Cognition and Instruction*, *18*(4), 495-523. http://www.jstor.org/stable/3233891

Jensen, A. (1978) The nature of intelligence and its relation to learning. *Melbourne Studies in Education, 20*(1), 107-133. https://doi.org/10.1080/17508487809556119

Lesh, R., Post, T. R., & Behr, M. (1987). Representations and translations among representations in mathematics learning and problem solving. In C. Janiver (Ed.), *Problems of representations in the teaching and learning of mathematics* (pp. 33-40). Lawrence Erlbaum.

OECD PISA (2021). https://www.oecd.org/pisa/pisaproducts/pisa-2021-mathematics-framework-draft.pdf

OILER (2022). Bul game. https://en.oiler.education/bul. Accessed: 2025-5-5.

Rushton, S. J. (2018). Teaching and learning mathematics through error analysis. *Fields Mathematics Education Journal*, *3*(1), 4. https://doi.org/10.1186/s40928-018-0009-y

Smullyan, R. M. (1987). *Forever undecided: a puzzle guide to Gödel*. A.A. Knopf.

Vaci, N., Edelsbrunner, P., Stern, E., Neubauer, A., Bilalić, M., & Grabner, R.H. (2019). The joint influence of intelligence and practice on skill development throughout the life span. *Proceedings of the National Academy of Sciences, 116*(37) (pp. 18363-18369). Michigan State University. http://doi.org/10.1073/pnas.1819086116

de Winter, J.F.C. (2013). Using the Student's t-test with extremely small sample sizes. *Practical Assessment, Research, and Evaluation*, 18, Article 10. https://doi.org/10.7275/e4r6-dj05

Zan, R. & Martino, P. D. (2017). *Insegnare e Apprendere Matematica con le Indicazioni Nazionali.* Giunti Editore.

Corresponding Author Contact Information:

Author name: Luigi Bernardi

Department: Doctorado Interinstitucional en Educación

University, Country: Universidad Distrital Francisco José de Caldas, Colombia

Email: l.bernardi@liceo-orazio.edu.it; lbernardi@udistrital.edu.co

Please Cite: Bernardi, L. & Manghi, R. (2025). Knight or knave? Description and evaluation of a programme for the introduction of logic at primary school. *Journal of Research in Science, Mathematics and Technology Education, 8*(SI), 195-218. DOI: https://doi.org/10.31756/jrsmte.418SI

Conflict of Interest: The authors declare that the research was conducted in the absence of any commercial or financial relationship that could be construed as a potential conflict of interest.

Data Availability Statement: Data available upon request.

Ethics Statement: This study obtained institutional review board approval prior to conducting any research related to this study.

Author Contributions: All authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.

Received: January 24, 2025 ▪ Accepted: May 27, 2025