

# Facing the Fear of Failure: Risk and Attempt in Mathematics Education

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## Abstract

Building on the Theory of Didactic Situations and the Theory of Objectification, this study investigates the role of risk within mathematical communities of practice, focusing particularly on how students' aversion to risk influences their learning processes. We introduce the concept of "attempt" as a specific form of risk-taking within mathematics education, defining the "propensity to attempt" as students' recognition of the importance of learning through trial and error, coupled with their capability to manage incorrect attempts on both mathematical and emotional levels. Motivated by the hypothesis that a higher propensity to attempt correlates positively with the development of certain mathematical skills, this qualitative research explores teaching practices that may either promote or hinder this disposition. The methodological framework involves collecting and analyzing student interviews and written productions from tasks related to mental calculation and early algebra, providing insights into how risk is perceived and managed in classroom settings. Preliminary findings suggest that environments encouraging risk-positive attitudes through devolution, knowing, and explicit discussions about learning and performance significantly facilitate students' mathematical exploration.

**Keywords:** Mathematics education, Risk and attempt, Mental Calculation, Theory of Didactical Situations, Theory of Objectification.

# Enfrentando o Medo do Fracasso: Risco e Tentativa na Educação Matemática

## Resumo

Com base na Teoria das Situações Didáticas e na Teoria da Objetivação, este estudo investiga o papel do risco em comunidades matemáticas de prática, focando especialmente como a aversão ao risco por parte dos estudantes influencia seus processos de aprendizagem. Introduzimos o conceito de "tentativa" como uma forma específica de assumir riscos na educação matemática, definindo a "propensão à tentativa" como o reconhecimento, por parte dos estudantes, da importância de aprender por tentativa e erro, aliado à sua capacidade de lidar com tentativas incorretas tanto em nível matemático quanto emocional. Motivada pela hipótese de que uma maior propensão à tentativa está positivamente correlacionada ao desenvolvimento de certas habilidades matemáticas, esta pesquisa qualitativa explora práticas docentes que podem promover ou inibir essa disposição. A estrutura metodológica envolve a coleta e análise de entrevistas com estudantes e produções escritas provenientes de tarefas relacionadas ao cálculo mental e à álgebra inicial, fornecendo indícios sobre como o risco é percebido e gerenciado em contextos de sala de aula. Resultados preliminares sugerem que ambientes que incentivam atitudes positivas frente ao risco por meio de devolução, conhecimento e discussões explícitas sobre aprendizagem e desempenho facilitam significativamente a exploração matemática dos estudantes.

**Palavras-chave:** Educação matemática, Risco e tentativa, Cálculo mental, Teoria das Situações Didáticas, Teoria da Objetivação.

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## Enfrentando el Miedo al Fracaso: Riesgo e Intento en la Educación Matemática

### Resumen

Basándose en la Teoría de las Situaciones Didácticas y en la Teoría de la Objetivación, este estudio investiga el papel del riesgo dentro de las comunidades matemáticas de práctica, centrándose especialmente en cómo la aversión al riesgo de los estudiantes influye en sus procesos de aprendizaje. Introducimos el concepto de "intento" como una forma específica de asumir riesgos en la educación matemática, definiendo la "propensión al intento" como el reconocimiento por parte de los estudiantes de la importancia de aprender mediante ensayo y error, junto con su capacidad para gestionar los intentos incorrectos tanto en términos matemáticos como emocionales. Motivados por la hipótesis de que una mayor propensión al intento se correlaciona positivamente con el desarrollo de ciertas competencias matemáticas, esta investigación cualitativa explora prácticas docentes que pueden promover o inhibir esta disposición. El marco metodológico implica la recopilación y análisis de entrevistas a estudiantes y producciones escritas derivadas de tareas relacionadas con el cálculo mental y el álgebra temprana, proporcionando indicios sobre cómo el riesgo es percibido y gestionado en contextos escolares. Los hallazgos preliminares sugieren que los ambientes que promueven actitudes positivas frente al riesgo mediante la devolución, el conocimiento y discusiones explícitas sobre el aprendizaje y el desempeño facilitan significativamente la exploración matemática de los estudiantes.

**Palabras clave:** Educación matemática, Riesgo y intento, Cálculo mental, Teoría de Situaciones Didácticas, Teoría de la Objetivación.

### INTRODUCTION

The concepts of risk aversion and ambiguity aversion have been widely explored in the literature (Arrow, 1965; Pratt, 1964), particularly in relation to decision-making biases that can lead to suboptimal choices in work and economic settings (Kahneman & Tversky, 1979). Recent research extends these insights to social contexts (Bohnet & Zeckhauser, 2004) and examines how risk perception influences learning processes (Callander & Matouschek, 2019).

Research in Mathematics Education (ME) has developed well-established and interconnected theories (Asenova et al., 2020) that can provide a useful framework for examining the concept of risk. In this study, we focus on two fundamental theories: the Theory of Didactic Situations (TDS; Brousseau, 2002) and the Theory of Objectification (TO; Radford, 2021). Although these theories have not previously been used to study risk, they offer valuable frameworks for interpreting the phenomenon from a novel and complementary perspective.

After outlining the key points of TDS and TO, we examine the concept of risk through the lens these theories provide, identifying Dweck's work (1995, 2000)—which explores the relationship between learning and performance—as a useful reference for situating the issue within a broader perspective. We then introduce the idea of “attempt” as a particular type of risk undertaken in the mathematics classroom. The scope of investigation is subsequently narrowed to arithmetic and mental calculation, domains particularly suited to this analysis. Studies on mental calculation emphasize the importance of tolerance to risk and experimentation in the learning process (Threlfall, 2002). Finally, we present and discuss an

educational path that spans arithmetic and algebra, aimed at cultivating risk tolerance in the context of ME.

## THEORETICAL FRAMEWORK

### The Theory of Didactical Situations

We recall some key points of the Theory of Didactical Situations (TDS), developed by Guy Brousseau starting in the 1970s (Brousseau, 2002). The TDS presents concepts and models intended to interpret and to allow the diffusion and evolution of mathematical knowledge, identifying the teacher's and student's roles in the different phases of this process. Within this theory, a didactical situation has two chief characters: the teacher who wants to teach a mathematical content and the student. The two actors are linked by the Didactic Contract (DC), which shapes their roles in the classroom and in relation to knowledge. In other words, the DC determines, explicitly for a small part, but above all implicitly, what each partner (the teacher and the student) is responsible for managing and what they will take care of in one way or another. In this sense it is a contract, with specific clauses.

One of the main situations described by Brousseau is the *adidactical situation*: the student is immersed in an environment, to be intended in a broad sense and called a *milieu*, in which they can act and receive feedback on their actions. The teacher creates such a milieu, where the knowledge to be learnt appears as an answer to a "game" played by the student. In fact, the teacher poses a problem but also makes a *devolution* of this problem, that is the problem is taken over by the students as their own and they feel engaged in the solution of the problem. So, devolution consists of putting the student into a relationship with a milieu from which the teacher is able to exclude themselves, at least partially. In this model, the devolution ensures the conditions for *adaptation*, and the institutionalization ensures the conditions for acculturation (Mangiante-Orsola et al., 2018).

Situations that allow the student's adaptation require a certain time; students must be able to make various attempts to explore and represent the problem, drawing conclusions from both failures and successes. "The uncertainty into which they are plunged is a source of both anguish and pleasure. The reduction of this uncertainty is the aim of intellectual activity and is its driving force" (Brousseau, 2002, p. 45). But the possible anguish cannot be countered by providing a mechanical method to reach the solution: "having transformed sufficient and particular answers into methods that give the answer every time, destroys the uncertain nature of the situation, which then loses its interest" (*ibid.*, p. 45). In other words, the teacher must resist the temptation to relieve the student from the anguish experienced during an activity involving uncertainty. While such intervention might provide emotional reassurance, it may also place the student in a position of dependence on the teacher, and

it risks “removing all meaning from the situation that instigates it [*i.e.*, the anguish]” (*ibid.*, p. 45).

Brousseau and Warfield (1999) describe the situation of a student, Gael, who—when faced with any objection—becomes uncertain about his answers, even if they seemed correct to him at first. The student seems to want to avoid confrontation and evades any conflict by taking refuge in a position of dependence. This obviously affects his mathematical knowledge:

In the area of knowledge, there is, in effect, an attitude where dependence offers the non-negligible benefit of a security: knowledge is always somebody else’s knowledge which one has only to appropriate; thus, one eliminates the risk of having oneself put into question in a debate about truth. There is no need to offer any reason for what one takes for truth other than the invocation of the authority to whom one refers. (Gaël says ‘what I was taught’, ‘what the teacher says I have to do’) (Brousseau & Warfield, 1999, p. 19).

Such an attitude opposes the development of personal mathematical knowledge as a result of engagement with reality, leading to a preference for a form of knowledge based on the reproduction of standard models. This kind of learning situation gives “no opportunity to the student to make or attempt a decision—thus to learn” (Brousseau & Warfield, 1999, p. 20).

In Gael’s case, the *devolution* occurs by transforming the problem posed by the teacher into a bet between two players (the tutor and the student). This encourages Gael to proceed through trial and error to reach the result. The authors remark that initially Gael is only trying his luck in this process of trials and errors, without undergoing a mindful process, but little by little efforts at reasoning begin to appear.

It is important for our study to underline here the quite insightful observation of two different types of students, or rather two different reactions observed in students when they proceed through trial and error. These reactions can only be observed in an adidactical situation, when the student has accepted an activity for which they take responsibility and where they can know on their own whether they have succeeded or failed. “In acknowledging that the results do not conform to their predictions, certain children turn pale and worry; they read this result as a personal failure and look discouraged and guilty” (Brousseau & Warfield, 1999, p. 84). For others, there is an almost opposite reaction: “something unexpected and interesting is happening; personal failure is overcome and minimized in favor of curiosity and an opening towards the exterior” (*ibid.*, p. 84).

## The Theory of Objectification

The Theory of Objectification (TO) is a dynamic system driven by dialectical relationships between *knowledge*, *knowing*, and *learning* (Radford, 2021; Paradigma, 2024). As

stated by the author, it is a Vygotskian perspective on knowing and becoming in mathematics teaching and learning. The theory of objectification is an epistemological theory, but it does not limit itself to describing and interpreting learning phenomena: it also aims to suggest how pedagogical practices and the entire classroom environment can be transformed into a place “where students can encounter cultural knowledges and voices in deep conceptual ways while at the same time making the experience of collective life, solidarity, plurality, and inclusivity” (Radford, 2021, p. 5).

In traditional teaching, the interaction between teacher and student is based on the recognition of the teacher as the authority and the subjection of the students to the teacher’s authority (Radford, 2021), and what occurs is a transmission of knowledge. In constructivist teaching, on the other hand, what we find is the ethics of the student’s autonomy, their freedom, and their right to construct their own knowledge.

The TO does not align with either of these two views, but rather engages in dialogue with both, as it proposes a different vision of the educational project:

it posits the goal of Mathematics Education as a political, societal, historical, and cultural endeavour aimed at the dialectical creation of reflexive and ethical subjects who critically position themselves in historically and culturally constituted mathematical practices, and who ponder new possibilities of action and thinking. As a result, the focus is not on the mathematical content alone; the focus is not only on knowing but also on becoming (Radford, 2018, p. 2).

Our encounter with and appropriation of culturally and historically constituted knowledge (e.g., mathematical, scientific, aesthetic, legal, etc.) is what is called *objectification* (Radford, 2018). Learning is the result of this process of objectification, but it must always be viewed as partial and evolving. The processes of *subjectification* are also introduced, that is, processes where “co-producing themselves against the backdrop of culture and history, teachers and students come into presence [...] To come into presence is a dialectical movement between culture and the individual” (Radford, 2018, p. 6). With the process of subjectification, students come to “occupy a space in the social world and take a perspective in it” (*ibid.*, p. 6).

In TO, what makes learning possible is human, sensuous, practical activity. *Joint labour* (in German: *Tätigkeit*) is the main ontological category of TO. It comprises “notions of self-expression, rational development, and aesthetic enjoyment” (Donham, 1999, cited in Radford, 2021, p. 38). It is important here to highlight the fundamental role of *self-expression*: what characteristics must a socio-cultural context have in order for an individual to express themselves? Conversely, what are the practices that limit this possibility of expression?

Let us further clarify some of the concepts at play: consider what happens within a transmissive teaching-learning activity. The teacher chooses a problem and shows how to solve it. Then the teacher lets the students answer similar questions. It can be argued that

there is, within this activity, a process of objectification. There is, in fact, an encounter with cultural-historical knowledge. But the encounter is very limited (Radford, 2021): the division of labour that underlies the activity (the teacher says; the students listen and imitate) leads to a very restricted form of production of knowledge, where appropriation is not granted. Imitation is indeed a fundamental process, in which knowledge is observed and replicated, even if only partially. While imitation is a key aspect of learning in general (Lyons and Berge, 2012), it plays a particularly significant role in communities where there is no didactical transposition of knowledge, and teaching-learning is directly embedded in social practices (Bagni et al., 2007).

In fact, if the personal and cultural context allows or even encourages it, the individual may take liberties in this practice of imitation. More precisely, the individual begins to integrate and elaborate their observation with internal resources, engaging in an active form of learning that involves their prior experiences. This act is defined by Radford as *knowing*, which is the individual elaboration of the knowledge and its consequent transmission by to the community. However, this freedom in the practice of imitation inevitably carries certain risks. In the process of knowing, the individual, in fact, returns a different knowledge to the community, taking the risk of making mistakes, being challenged, judged and devalued. To echo Radford's words, the processes of objectification are social processes in which the individual becomes aware of a way of doing and thinking within a historical-cultural system. However, this process is not one of submission or subordination; rather, it fundamentally requires a critical perspective and space for dissent and change in practice. Whether this space exists, and how large it is, clearly depends on the context in which one is situated.

To facilitate the process of objectification, it is therefore necessary to reflect on how much space the individual has and how the risk of a critical perspective and dissent is perceived by the individual and by the group. Since there are multiple ways to understand a group and a community (D'Amore, 2015) it is not generally true that individuals should be encouraged to take risks in every type of environment, but in some contexts—like a learning environment—it is beneficial for individuals to be tolerant of risk and ambiguity (Callander & Matouschek, 2019).

## **PERFORMANCE, RISK AND ATTEMPT**

The distinction that Brousseau and Warfield (1999) make between the two types of behaviors observed in students when faced with failure is really similar to the one made by Dweck (2000), who identifies a distinction between *mastery-oriented* students and *helpless* students. The term “helpless” is used to describe some students' perception regarding failure, a viewpoint according to which—once a failure has occurred—the situation escapes the control of the student who believes they can no longer do anything (Dweck & Reppucci,



1973; Dweck, 1975). The definition of helpless is then extended to include all reactions exhibited by these students in the face of failure: low regard for their own intelligence, lowered expectations, negative emotions, reduced persistence, and a drop in performance (Diener & Dweck, 1978). Conversely, the term mastery-oriented students has been used to refer to the ability to approach failure in a positive and proactive manner, remaining focused on learning despite the existing difficulties.

Dweck investigates what generates this evident distinction in the dynamics reported above, and her answer is formulated in terms of *goals*: mastery-oriented students have different goals compared to helpless students. The goal of mastery students is a learning goal, reflecting the desire to acquire new skills and understand new things: a desire to become more intelligent. The goal of the helpless, on the other hand, is a performance goal. This goal is about earning positive judgments of one's abilities and avoiding negative ones. Elliot (1999) likewise states that performance goals focus on the demonstration of competence relative to others, whereas mastery goals focus on the development of competence. In other words, when students pursue performance goals, they are, too, concerned about their level of intelligence: they want to appear smart (to themselves or others) and avoid mistakes. As can be seen in both definitions, the concept of intelligence is present: what is important here, however, is not the general definition of intelligence, but the meaning each student attributes to this word: for the mastery-oriented, it is something mutable that improves over time, while for the helpless, it is something static and definitive. This is why Dweck discusses Self-Theories, that is, the theories each individual has about their own self. Brousseau and Dweck would agree that *performance-oriented* individuals (such as Gael in the situation described above) particularly benefit from an adidactical situation where the teacher effectively enables the student to take ownership of the problem and encourages risk-taking.

As Dweck herself specifies, the duality “performance vs learning”—in a deeper sense—does not correspond to two types of students, but rather represents an internal movement of the individual that leads one component to prevail over the other. Clearly, which component prevails is domain-specific and dependent on the reference community (D’Amore, 2015). The more judgmental and performance-based the community is, the less an individual will feel free to exercise *knowing* (Radford, 2021) for fear of being devalued. This extends also to the family environment: students are not only subject to an assessment in class, but also to a meta-assessment at home (an assessment on the assessment). Consequently, either they will avoid knowing altogether, becoming passive in front of teaching, or they will distort its meaning: instead of being the product of an antithetical process borne by the individual, it is a rephrasing of knowledge to return to the community what the community is expected to want. That is, in a DC where performance prevails, the student gives back to the teacher what they think is expected by the teacher. Since learning is an internal process that cannot rely on expectations, learning is not so much based on the proper function-

ning of the contract, but on taking the risk of “breaking the contract” (Brousseau, 2002, p 32). D’Amore et al. (2023, p. 98) invite the teacher to “create the social, affective, and didactic conditions of the breaking of the DC in order to encourage the student to rely only on himself to construct, with or without others, his own meanings”.

It is notably insightful that Sharma (2015) reports that, according to a number of writers (e.g., Ames & Archer, 1988), tolerance for risk is directly related to students’ perceptions about their own goals (performance or learning). Atkins et al. (1991, p. 297) also define risk in an educational context as “the preparedness of a student to attempt to answer a question when not certain of the result”. Abercrombie et al. (2022), drawing on—like Radford—a Vygotskian perspective (Vygotsky, 1978), connect academic risk-taking to learners’ performance within their zone of proximal development. We emphasize that also for Radford *knowing* is relative to the zone of proximal development.

We introduce a broader definition: the personal aversion to risk in a mathematics classroom setting is indicative of how difficult it is for the individual to break some clauses of the DC, no matter how senseless they may appear (D’Amore et al., 2023). This definition encompasses those by Dweck, Sharma, and Atkins. For example, in relation to Atkins et al., if a student has a clause in the DC—explicit or perceived—that making a mistake is wrong, they will be reluctant to attempt to answer a question if they are not sure of the answer.

There are various studies that seek to correlate and find causal relationships between risk aversion and some specific individual abilities. A result obtained in Butler et al. (2013) positively correlates risk tolerance with good intuitive abilities of the individual. Mazza and Gambini (2023), by analyzing the results of the regional competitions from all the regions, bring to light interesting quantitative evidence on how lower risk aversion correlates with better performance in certain mathematical contexts.

### **Definition of Attempt propensity**

Let us focus on a specific category of perceived risk in the classroom, namely the risk associated with the attempt. We define *propensity to attempt* as the student’s awareness of the importance of learning based on trials and errors and—consequently—their ability to manage, both at the mathematical and emotional level, incorrect attempts. We highlight that this definition is strictly included in the concept of risk defined previously and not everything that is a risk is necessarily an attempt: answering a multiple-choice question is certainly a risk, but not an attempt, because the act does not involve the possibility of being able to perform it again based on previous acts.

The aversion or propensity to attempt can manifest in various forms in the school context. In TDS, attempts typically occur within an adidactic situation and can be viewed as an adaptation to the milieu. As Brousseau emphasizes, adaptation requires time and ac-



tive participation in the new environment created by the teacher. However, as we will see in some of the interviews conducted with the students involved in the experiment, it may happen that the student perceives the attempt as risky due to the fear of being judged by the teacher. In this case, the student sees the attempt as a potential breach of the DC. Alternatively, as in the case of Gael, a form of “laziness” may emerge, where the student feels safer waiting for someone else to solve the problem, thereby avoiding personal engagement. In both cases, the phase of devolution is crucial. The student must take ownership of the problem to be solved. Being willing to make an attempt means engaging fully in the task and taking the necessary steps to adapt to the situation.

Just as in the TDS the attempt occurs in an adidactic situation, so in the TO it occurs during a phase of activity, of joint labour. The joint labour corresponds to a social process that is at the same time a process of objectification—as the students are encountering something culturally relevant—and subjectification—as the students are coming into presence and positioning and being positioned in a mathematical practice. In TO, the attempt is equivalent to knowing, as it takes place during the ongoing exchange between subjectivation and objectification. The more immersed a subject is in the activity, the more inclined they are to attempt, especially when working alongside others who are engaged in the same task. If a student perceives sharing the knowledge they have acquired with others as risky, it indicates that the community is not truly engaging in joint labour. In fact, it may happen that the teaching-learning activity is so weak that students lack the space to express themselves, failing to recognize themselves in the outcomes of their efforts (Radford, 2018).

This study, which is in its early stages in this article, first aims to verify to what extent it is possible to refer to “propensity to attempt” as a characteristic of an individual within a learning community. Building on this, it explores whether there are teaching practices capable of promoting—or at least not inhibiting—this disposition, hypothesizing a correlation between “propensity to attempt” and the development of certain mathematical skills. The study will adopt a qualitative research methodology, drawing heavily on TDS and TO. In particular, in this article we will focus on the sphere of mental calculation within the context of early algebra (Radford, 2014), which proves to be an ideal environment for exploring the questions we have posed.

## **Mental Calculation**

We shall now direct our attention on numerical attempt in the context of mental and arithmetic calculation. Mental calculation certainly plays a foundational role in an individual’s mathematical abilities, and it can be said that a student with a high level of mental computation has a high level of mathematical reasoning (Gürbüz & Erdem, 2016).

Craig (2009) compares what is referred to as strategic thinking in Mathematics Education (ME) with certain studies from developmental psychology, particularly in relation to mental calculation. The author argues that the concept should take into account aspects such as the role of memory and both conscious and unconscious mental processes. Indeed, Craig (2009, p. 3) reports how “another interesting subtlety in the psychological model of strategic thinking—which is lost in current research in ME—is the importance of performance-related factors”, which potentially include social and affective goals (indeed part of the DC), and that performance orientation can be seen to encourage short-term mathematical behaviour which does not promote long-term arithmetic development. This view perfectly aligns with our theoretical framework.

Threlfall (2002) argues that the royal road to mental calculation is *flexibility*, where flexibility is defined as in Thompson (1999), who—among four attributes that assist in the development of flexible mental calculation—identifies “positive attitudes, such as the confidence to ‘have a go’ and not being put off by immediate lack of success”. Flexibility cannot be taught as a skill and it will arise consequentially through the emphasis on considering possibilities for numbers, *i.e.*, as individual construction through a process of trial and error. Students who fear mistakes may struggle to develop flexibility in calculation. If students are unsure about their knowledge or feel they must always choose the “correct” strategy, they may hesitate or fear making mistakes. Teaching fixed mental calculation strategies can limit students’ flexibility and make them rely on memorized procedures, pressing the students to perform well rather than focusing on deep mathematical understanding (Threlfall, 2002).

Libertus et al. (2011) find that children’s Approximate Number System (ANS) acuity correlated with their math ability, even after accounting for differences in age and verbal skills. Similarly, Mazzocco et al. (2011) reveal that the ability of preschoolers to estimate numbers (ANS precision) predicts their math performance at six years old, independent of other cognitive skills. An additional result is presented in Liang et al. (2022), where it is shown that working on the ANS improves arithmetic skills, and vice versa. To understand the role of ANS in our framework, it is important to acknowledge that Manghi and Bernardi (forthcoming) show not only that ANS is positively correlated with risk tolerance, but also working on ANS accuracy seems to lower risk aversion.

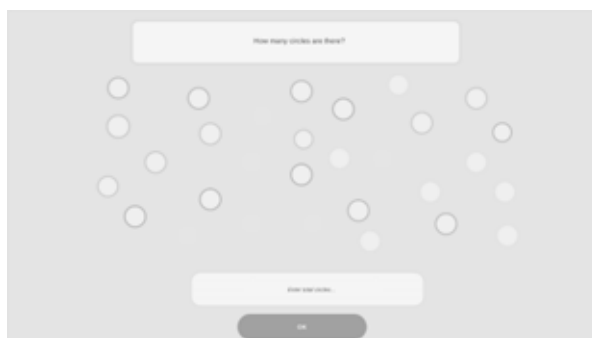
## **THE EDUCATIONAL PATH**

This section describes an educational path aimed at promoting a positive attitude in students towards attempts through mental calculation and early algebra. Each activity has been structured following the example of the phases of the development of a didactic situation as in Brousseau (2002). The activities were conducted in two fifth-grade classes of an Italian primary school in Bogotá, Colombia, with students aged 10 to 11.

## WarmApp

Each activity begins with a round of the *WarmApp* game (OILER, 2023). The game promotes skills consistent with ANS and System 1 in general. For example in Fig. 1 the player must determine how many balls are in a given set by observing it for a very brief period.

**Figure 1** : Game screen of WarmApp



**Source:** Author's archive

We chose to begin each activity with this game because—as noted at the end of the previous section—it provides a way to work on risk-taking while working on ANS.

The students identified the skills solicited by the game as visual and cognitive, particularly highlighting attention and memory, which were necessary because the image disappeared quickly. Actually WarmApp works on *flexibility*. The intent to fix the image in memory seems to stem from the need to obtain an exact answer rather than relying on one's intuition, something to which students are not accustomed in a mathematics lesson.

## First activity: Guess the secret number

The aim of the first activity is to introduce students to the concept of attempt through a game. The game involves two players: one player thinks of a number between 0 and 100, while the other attempts to guess it. After each guess, the first player responds with “greater” or “smaller” until the correct number is found. The process occurs mentally, without written aids. After a phase of pure play (Brousseau, 1980), the next stage—Devolution of Responsibility and Causality—introduces a more defined and conscious goal, where winning means guessing in the fewest attempts. To keep track of the number of attempts, each pair had to use a sheet of paper to record their plays.

A tournament was therefore held to promote a reflection on the implicit strategies developed by students and to enable them to compare their approaches with those of their peers. Subsequently, following the final phase of devolution in the didactical situation, the winning strategy was discussed, first through a written production and then through an oral debate. In other words, victory is no longer attributed to luck but to the ability to make informed choices.

### Second activity: The number eating monster

The country of Robinson, named after Julia Robinson, is introduced to the class as an imaginary land inhabited by various tribes of monsters. These creatures have a unique characteristic: they eat numbers. When a monster eats a number, the monster itself transforms into a number. In the first part of the activity, various tribes of monsters are presented. The goal of the class is to determine the transformation rule followed by each tribe; in other words, the goal is to understand  $f(n)$ , first by intuitively grasping it on an arithmetic level and then explicitly formulating it algebraically. To achieve this, the class must proceed by attempts: a number is proposed, and the teacher reveals the number into which the monster has transformed it, until the rule becomes clear. This first phase is therefore entirely focused on mental calculation.

We asked how to express mathematically the event of the monster eating a number. For example, how to communicate the event that the tribe called XEN (a tribe that by eating a number transforms into its next number) ate the number 5 and transformed into 6. The writings proposed by the students were varied and interesting, as  $XEN \cup 5 = 6$ , where “ $\cup$ ” is a symbol that indicates a mouth. Other symbols used have been—among others—the  $+$  and a crocodile face. Finally, a student proposed the use of parentheses  $()$ . It was highlighted that parentheses are the most commonly adopted symbol by the international mathematical community, making them a useful choice to promote a shared language (acculturation). The class agreed on the expression  $XEN(5) = 6$ .

At this stage, it was essential to transition from a specific case to a general representation of the rule governing the monster’s transformation. Building on the previous lesson, where the variable was introduced, students were encouraged to explore different representations until they arrived at the expression  $XEN(n) = n + 1$ .

The class is then invited to work in pairs to create their own tribe, formulating a rule for their tribe that their classmates would have to deduce.

### Third activity: solving equations by trial

The third activity involves a game where single-variable linear equations are scattered across a floor together with numbers. The purpose of the game is to place a suitable number over the variable  $n$  of an equation to satisfy the equality. It is emphasized with the class that in the case of an equation where  $n$  appears more than once, it must always have the same value.

Following the concept-process duality (Gray & Tall, 1994), a fifth-grade student perceives an equation as a concept rather than a process. Lacking algebraic manipulation skills, the only available method they have is to substitute the variable with a natural number and mentally verifying the equality. If the equality is false, the number is removed, but the at-

tempt has contributed to giving additional meaning to the concept (i.e., the equation). Significant attention is given to the words *all*, *at least one*, *none*: some equations are solved by all numbers, like  $2 + n = n + 2$ , others by at least one number, and others by no number, like  $n - n = 3$ .

Generally speaking, students begin to make conscious decisions about the values to be assigned to  $n$  because they start to recognize—in an equation of the kind  $f(n) = g(n)$ —a connection between  $n$  and  $f(n)$  and  $g(n)$ , acknowledging the causal relationship between their choices and the outcomes obtained. As a final step in the activity, students are asked to create their own equations for their classmates to solve.

## DISCUSSION

Let us now qualitatively analyze certain dynamics observed in the classroom, complementing these observations with data collected through interviews and questionnaires. The interpretation of this empirical evidence will be framed within the theoretical background introduced earlier, based on TDS (Brousseau, 2002) and TO (Radford, 2021). In particular, attention will be directed toward students' propensity for risk-taking and attempt in the context of ME.

### Didactic Contract and implications for risk taking

At the end of the first activity *Guess the Secret Number*, interviews were conducted with students to explore their perspectives on risk and attempt. We present the most significant excerpts.

Interview with L.

**Teacher:** "Are you afraid that by giving a quick response you might make more mistakes?"

**Student:** "No, because teacher B. [a teacher] always tells us that we should think before we speak, and so I got used to it [explicit clause of the DC]. He tells us it's better to think before saying something." [...]

**Teacher:** "You said that it's right to think before you speak. What does a person who doesn't do this risk, who responds quickly? What's the danger? What might they be afraid of?"

**Student:** "In math or in life?"

**Teacher:** "Good question. Both, let's start with math."

**Student:** "In math, you risk *flunking the subject* [being postponed], because you get a low grade."

It is made explicit that answering quickly is a mistake when considered in relation to the performance to be achieved (Dweck, 2000), namely not running the risk of receiving a low grade.

**Student:** "In life, it's because other people's esteem for you lowers [...]"

This statement perfectly aligns with Newby & DeCamp (2014).

**Teacher:** “ [...] In your opinion, the kind of reasoning we practiced today, quick and intuitive, is it an acceptable way of thinking in mathematics?”

**Student:** “Today yes, because at the beginning you told us that it wasn’t necessary to do calculations.”

The “yes” emerges as the exception that proves the rule: mathematics is primarily associated with calculations and *System 2* (Kahneman, 2011), rather than the ANS. This response shows, however, how readily students are able to adapt to changes and breaches in the DC, and that risk aversion is more situational than an inherent characteristic of the individual (Cuevas *et al.*, 2018).

**Teacher:** “... You told me about two problems with giving quick responses: one is linked to the risk of a low grade, while the second example you gave me is about the impression we make on others. Let’s make an example: you are doing geography and the teacher asks you what the capital of Australia is and you don’t know. You then start saying various cities, like Bogotá, Paris, Rome. What would the teacher think of you?”

**Student:** “That I don’t think before I speak [resumes the clause of the DC instinctively, with the exact same words used by teacher B.]. Actually, two things. The first is that at least I’m trying to answer, and also that I might be lacking a bit in my studies.”

**Teacher:** “That’s interesting, because you’ve said it’s both a positive and a negative thing.”

**Student:** “I really like school, I really enjoy learning [learning oriented]. If I were the teacher, I would think it’s a positive thing.”

It is important here to highlight that the student, after impulsively invoking a clause of the DC, engages in reflection on that very clause and identifies positive aspects in the proposed approach. Although we shall return to this point in greater detail shortly, it seems that an explicit reflection on the DC may facilitate the breaking of clauses that are not endorsed. The learning-oriented attitude appears to conflict with the fear of being judged by the teacher.

Interview with E.

**Teacher:** “When L. asks a question or when I ask a question and call you out, how do you feel about responding?”

**Student:** “I feel a bit nervous because if I say something wrong or absurd, then others start laughing, so I get anxious and don’t respond, and I freeze.”

**Teacher:** “So, are you afraid of the teacher or your classmates?”

**Student:** “I’m afraid of making mistakes.”

**Teacher:** “And why are you afraid of making mistakes? What does making a mistake mean to you?”

**Student:** “Saying something that’s not right, like  $2 \times 4 = 9$ . [The student tries to explain themselves but can’t manage and becomes anxious.]”

**Teacher:** “Do you think making mistakes can also be a way to learn?”

**Student:** “No, because if you make too many mistakes then you don’t learn. When I make a mistake, I get anxious because everyone is looking at me.”



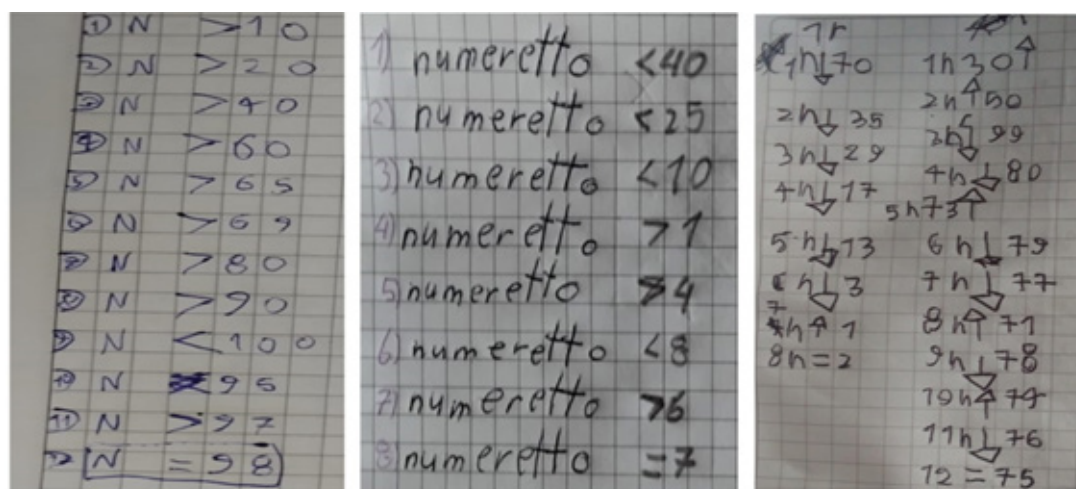
In this second interview, the student's self-awareness reveals their risk-averse tendency, where error is perceived solely as a synonym for failure and thus something to be avoided (Zan and Di Martino, 2017).

Notably, after this interview—conducted by the main teacher, with whom the student has a good relationship—E. was very engaged in the next activity. The teacher noticed that she had assumed a behavior not in line with previous attitudes: she raised her hand, she was happy, she participated in shared conversations in which she usually remained silent. One could hypothesize that speaking explicitly about factors related to the attempt and risk could be helpful: indeed, since risk aversion is domain-specific, addressing it becomes more manageable: it requires the student to make an adjustment limited to the mathematical context rather than a fundamental change in their overall attitudes. The fact that explicitly discussing risk aversion and the potential disadvantages it entails can lead to a reduction in one's own aversion is perfectly in line with the findings by Jia *et al.* (2020).

### ***Devolution and Attempt as drivers of good Knowing practices***

In the first activity *Guess the Secret Number*, attempts play a central role, as students have no other strategy than to progressively test different numbers until they identify the correct one. The optimal strategy to guess the number in the fewest possible attempts is the bisection method. Anyway, during the game, two distinct approaches to proposing numbers emerge: on one hand, some students proceed cautiously, adopting a sequential approach, saying for example at the beginning in sequence the numbers 10, 20, 30, 40, until they find a “smaller” answer or—at a certain point in the play—saying the numbers decreasing exactly by one until they find the correct answer, e.g. 80, 79, 78. On the other hand, some engage in a more exploratory approach, experimenting with “unusual” numbers and freely jumping within the remaining interval. Even if it is difficult to discern whether the first approach is due to greater attempt aversion or lesser mathematical ability, one could argue that this behavior may be correlated to a risk-averse attitude, as players might perceive these choices as “normal” and less risky, preferring to advance by regular increments.

**Figure 2:** Guessing the number



Source: Author's archive

We emphasize that in this initial phase of the activity, a *devolution* occurs, as the game is perceived as enjoyable independently of the lesson and is played multiple times

by students without any external prompting. However, there is no *knowing*, as there is no subjective reworking or restitution of the activity to the community. Knowing occurs when students are asked to record the progression of the game in writing. They document the concept of the hidden number and the answer obtained from the other player at each step in their own preferred manner. As can be seen in Fig. 2, various were the names attributed to the variable. As an additional note, it is important to observe that some students were unable to mark the moves using the conventional symbols  $>$  and  $<$ , instead preferring arrows, as in the third example of Fig 2. This phenomenon was observed in eight working pairs across the two classes. On the one hand, the freedom to choose the variable name and, in some cases, even modify the greater-than and less-than symbols indicates that devolution has been successfully achieved. However, it is surprising that the greater-than symbol has not been fully objectified in fifth grade.

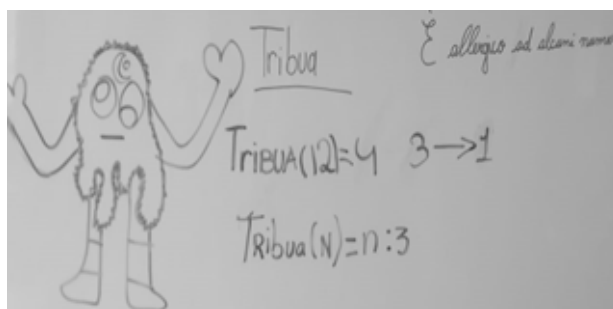
At the end, a discussion was opened to understand if and how the “secret number” game was connected to the WarmApp game. Among the concepts that emerged, “speed” and the element of “doubt” and “uncertainty” stood out, highlighting the importance of the concept of “attempt”. One student even talked about the “self-confidence” necessary to confront the uncertainty itself.

**Figure 3:** A new tribe



**Source:** Author's archive

In the second activity, *The Number-Eating Monsters*, just as in the first, attempting represents the only possible strategy for identifying the functions. The most relevant phase of the activity for our analysis is when the class is tasked with creating their own monsters (Fig. 3, left). In this phase, an important process of *joint labour* emerges—encompassing both *objectification* and *subjectification*—and, consequently, *knowing*, as students return the acquired knowledge to the community. The creation of a monster constitutes an immediate re-elaboration and sharing of the learned concepts, which are then subject to evaluation and discussion by the rest of the class. This evaluation considers not only the monster's algebraic properties but also its aesthetic aspects that students care about: we note that the monster in Fig. 3 has 6 legs which correspond to the coefficient of the variable.

**Figure 4:** A new tribe, allergic to certain numbers

**Source:** Author's archive

In this phase, the emergence of new cultural elements can generally take place, as was indeed observed in the classroom. Some students, for instance, chose to use division when defining the transformation rule for their monsters—a strategy not present in the “tribes” previously introduced by the teacher. However, they soon realized that division did not always yield an integer result and were aware that the activity had consistently been conducted with whole numbers. The solution they proposed to address this issue was to state that the monster was “allergic to certain numbers” (Fig. 4). The interpretation is remarkably fitting for conceptualizing the domain of a function that is not defined over the entire set of natural numbers. This new knowledge, introduced through the risk taken by a small group of individuals (using division and inventing the concept of a monster allergic to certain numbers), leads to the emergence of new cultural elements, which are then subject to institutionalization.

Another significant moment in the activity was when some students created a monster that, when eating a number  $n$ , transformed it into  $n \times n + n$ , and they then presented this rule to the class as a challenge to decipher. After a few attempts, the class correctly identified the function, but instead of expressing it in the form  $f(n) = n \times n + n$  (as originally conceived by the creators of the monster), they described it as  $f(n) = n \times (n + 1)$ . This discrepancy led to some difficulties, as the students who had designed the monster did not recognize their classmates' formulation as equivalent to their own. This phenomenon—where different yet equivalent representations of the same function were provided—occurred multiple times. The teachers excluded themselves from the discussion and after some time the students agreed that the writings were equivalent (*i.e.*, that the solution was correct). The act of knowing led to the emergence and discussion of an important duality between object and representation.

In the third activity, *devolution* occurs when students have to solve equations by trial and error (Fig. 5) and *knowing* occurs when students have to create their own equations (Fig. 5, 6).

**Figure 5:** Solving equations by trial and error.

**Source:** Author's archive

In the first devolution phase, after students have solved an equation, a verbal justification is requested. For example, faced with the equation  $n - n = 3$  a student justifies by saying “no number subtracted from itself can give a number different from 0”. Other interesting argumentations showed how the student had understood the meaning behind the equation: for example, in the case of the equation  $n + 3 = 2$ , it was emphasized that already 3 was larger than 2 and therefore the equation could not be solved with any natural number. Another interesting case was the equation  $n + n - 1 = 10$ , where the argument provided was “if we put 5 it becomes 9 which is too small but already if we put 6 it becomes 11 which is too big, so the equation is impossible”.

**Figure 6:** A new equation

**Source:** Author's archive

In the second phase of knowing, during which students create equations to share with their classmates (Fig. 6), a number of interesting dynamics emerged. In particular, one pair worked on changing the constant term in the equation  $n \times n + c = n$  (devised by the students themselves), finding a different solution for each value (Fig. 7).

**Figure 7:** A new equation

**Source:** Author's archive

Feeling free to engage in *knowing*—namely, to act without fear of judgment within the community of practice—proves crucial for entering the zone of proximal development. Indeed, several mathematically significant ideas emerged spontaneously, such as the use of the symbols *greater-than* and *less-than* (different from those commonly adopted, yet perceived by students as their own), the notation for function, the concept of domain, and the notion that changing the constant term in an equation yields different solutions. Developing these insights requires a certain tolerance for risk, a topic extensively discussed in the theoretical framework, which is further encouraged by the supportive environment in which these activities take place. In fact, making changes inevitably demands the courage to experiment, a factor that students value—whether consciously or subconsciously—as highlighted in the interviews.

In conclusion, we propose that *devolution*, followed by *knowing*, within a risk-positive setting, provides an ideal context for the emergence of new mathematical knowledge. Moreover, explicitly addressing learning, performance, and risk in the classroom can help mitigate aversion to ambiguity and foster a sense of freedom among students to explore mathematical concepts.

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