Chapter 9
Bul Game:
Playing With Knights and Knaves

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ABSTRACT

Bul Game is an online software designed to support didactic activities about logic and its connections with everyday language. The aim of the game is to make correct choices based on statements made by knights, who always tell the truth, and by knaves, who always lie. With small adjustments, the game can be used at all grade levels. As part of a programme to introduce logic at primary school, carried out in two second grade classes of an Italian school with many foreign students and one fourth grade class of a French school in Rome, Bul Game was used at the end of each lesson to consolidate and revise the concepts discussed in class. At the secondary-school level, Bul Game can aid the introduction of connectives and propositional logic, together with the characters of knights and knaves, which are a useful tool for dealing with proofs conducted with game semantics. Bul Game was designed and developed by Giulia Balboni, Luigi Bernardi, Mattia Sanchioni, and Jacopo Zuliani, and it is available for free at www.oiler.education/bul in English, French, and Italian.

INTRODUCTION

This chapter describes the features and possible use of *Bul Game*, a free online software designed to support didactic activities about logic at all school levels. The name *Bul* comes from the Italian pronunciation of Boole, the well-known English logician who is among the founders of modern mathematical logic. The characters of the game are inspired by the inhabitants of Smullyan Island (Smullyan, 1987), and indeed the aim of the game is to make correct choices based on statements made by knights, who always tell the truth, and by knaves, who always lie. As we will see, the game allows for increasing difficulties, which make it suitable for all grade levels. The author experimented with the game in two primary schools, specifically in two second-grade classes of an Italian school with many foreign-language students and a fourth-grade class of a French school in Rome, within a wider program to introduce logic at primary school.

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The purpose of this program is to introduce students to the concepts of negation, true and false statements, predicates, and connectives, always underlining the connections with everyday language. These concepts are not explicitly mentioned by Italian National Indications, it is worth noting that these Indications are not compulsory for the teacher to be followed. The characters of the knight and the knaves were used in all phases of the program. A more detailed description of this activity can be found at (OILER, 2021a) and (OILER, 2021c) and will be discussed in (Bernardi, in press). In this chapter, we only refer to some of the main features of the program in order to describe how *Bul Game* was used in this context.

Bul Game shares the same purposes of the aforementioned program; indeed, the activity with *Bul Game* aims to consolidate and revise the concepts discussed in class. The author believes that the program helps students' awareness of the structure of everyday language, without giving native speakers an overly privileged position.

BACKGROUND

The relationship between logic, language, and mathematical reasoning is a crucial one. With reference to it, Durand-Guerrier (2021) claims that "the difficulties related to questions of logic and language in the mathematics class are most often insufficiently taken into account by secondary and university teachers" (p. 2). This leads to the paradox that mathematical formalism—which should serve to clarify concepts—becomes instead an obstacle to students' learning. In relation to the same issue, Ferrari (2021) discusses similarities and differences between the communicative mechanisms inherent in natural language and the interpretation of symbolic notations in mathematical logic—with particular emphasis on connectives. Both authors advocate not only for an explicit introduction of logic in the mathematical school curriculum from the earliest years of primary school—thus refuting the idea that an understanding of basic logical concepts can be acquired automatically through standard mathematical teaching—but also for an appropriate use of symbolism.

In a multilingual context, the links between logic and language are connected to the fact that the structure of language can affect the thought process. "Logical connectives are one of the resources that a language uses to link and sequence ideas" (Edmonds-Whaten et al., 2021, p. 33) and their knowledge in the language of instruction seems to be the most important variable in deductive reasoning for bilinguals from different countries (Dawe, 1983). According to Durand-Guerrier et al. (2021) in a multilingual context "predicate logic can be used to unpack the logic of a given statement by identifying the logical categories, connectors, quantifiers, and their respective scopes" (p. 88). These concepts often remain implicit or are hidden through linguistic means depending on the language, and this can lead to ambiguities.

Natural languages are distinguished by the alphabet, by the words composed with the alphabet, and by the structure—that is, the rules that indicate how to arrange words to construct sentences. The fundamental characterization of a language may seem, at first glance, to be related to words only. However, structure also plays a fundamental role in a language because it clarifies how we express ourselves. Indeed, the importance of language rules is recognized. Moreover, as most of us have experienced, the translation of single words is not sufficient to interpret a sentence in a language unknown to us.

All the above references and considerations seem to corroborate our choice to introduce—in the teaching program as well as in *Bul Game*—formal structures for simple sentences containing predicates, negations, and connectives. On the one hand, this should slowly accustom the student to formalization

and to its translation in everyday language; on the other hand, it places native and non-native speakers on the same level, leaving comprehension of individual terms as the only possible problem.

When referring to logical games, we often find mention of puzzles or brain teasers, which are not explicitly linked to logic or mathematics, although this does not mean that they cannot have an educational value. For instance, Bottino and Ott (2006) analyze the use of computer mind games to develop strategic and reasoning abilities in primary school pupils. But there are also games explicitly linked to logic, starting from Carroll's classical Game of Logic (Carroll, 1887), designed to solve syllogisms. Coppola et al. (2019) construct the "World's game", where a child is asked to behave like a robot that only obeys certain commands; in this way, the game encourages children to construct a simple symbolic language. At a more advanced level we find reference to game semantics, which presents a game between two players, one of whom tries to prove a given statement while the other tries to refute the various demonstrative steps (Barrier, 2008; Bernardi, 2020). We will come back to game semantics later on.

The game presented here, *Bul Game*, is a game about predicate logic (but without quantifiers); it is accessible online, unlike the logic games just mentioned. It was designed and developed by Giulia Balboni, Luigi Bernardi, Mattia Sanchioni, and Jacopo Zuliani, and is available in English, French and Italian. The languages used during the game—that is, the languages in which the game can be played—have the characteristic of sharing the same structure, but not the same words. The attention paid to the structure of a sentence initiates a dialogue between syntax and semantics and is part of a process of "constructing and negotiating mathematical meanings" (Coppola et al., 2021, p. 9).

Several elements of classic online games have been used to "gamify" the logic and to motivate and engage students: captivating graphics, a beautiful landscape with a path to walk for as long as possible, the choice of the level of difficulty, competition with oneself (and with others, due to the presence of a ranking), autonomous rhythm and feedback (the game ends on the first mistake). Of course, its main feature is the help in learning reasoning by interpreting sentences which are made of words and symbols, spoken by two characters that pupils have come to know: the knight and the knave.

THE GAME DESIGN

The Language

The language used in the game is reminiscent of the classical language of formal logic, and the central theme is predicates. Predicates are a notation that indicate whether a property applies to one or more objects. For example, COLOR(x) is a predicate referring to a single object x (i.e., a unary predicate) that predicates the fact that "x is a color". So COLOR(red) is true, while COLOR(table) is false. It is important for students to always maintain the connection to everyday language and read such notation as, for example, "red is a color" and not as "color red". Moreover, we refer to a predicate as binary when it involves two objects. The relation between the two objects of a binary predicate can be symmetrical, such as SYNONYMS(x,y); unrelated, such as KNOWS(x,y)—that is, if x knows y, we have no information about whether y knows x or not (y could be a VIP!); or asymmetrical, such as MOTHER(x,y)—if x is the mother of y, surely y is not the mother of x.

The notation used to describe predicates is typical of logic, but it also recalls the ordinary notation used for functions, *i.e.* $f(x)$. In fact, each predicate is a function that associates to one or more objects the values 0 or 1 (*i.e.*, false or true). Another symbol that is used in the game is the negation symbol ¬,

which must be understood as *not*. For our purposes, the *not* is only placed in front of a predicate—and not in front of a more complex formula—to obtain a notation of the type ¬ANIMAL(tiger), which should be read as "a tiger is not an animal".

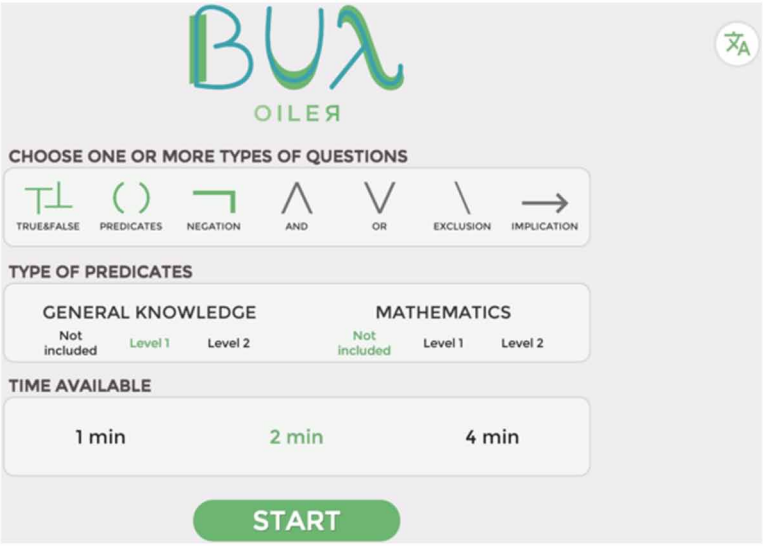
In the game, we find predicates referring to general knowledge, *e.g*. TREE(oak), as well as predicates referring to mathematical context, *e.g*. EVEN(3). We underline that statements such as $3 < 2$ are binary predicates, where the uncommon notation $<(3, 2)$ is avoided.

The classical logical connectives of conjunction $\wedge$ and disjunction $\vee$ are also used in the game, as well as the symbols of implication $\rightarrow$ and exclusion \, which will be discussed further later.

How to Play

The game is inspired by the characters of the knight and the knave. The game is played by pressing the A and B buttons—using the keyboard or clicking with the mouse—according to the clues given by knights and knaves. If you press the correct button, you score 1 point; if you make a mistake, the game ends. The aim of the game is to score as many points as possible in a given time. In the homepage menu you can choose how to set up the game choosing the type of questions given (true&false, predicates, predicates with negation, …), the type of predicates which will appear in the clues—general knowledge or mathematics—and the time available for your play (Figure 1).

Figure 1. In the homepage menu you can choose how to set up the game.

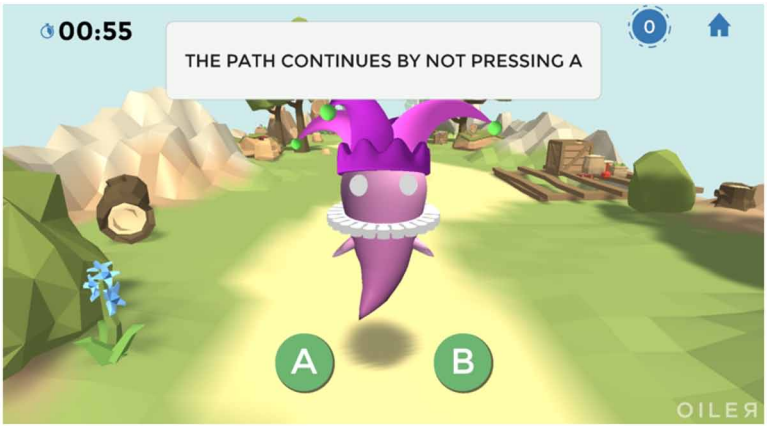


All questions that appear in the game are randomly generated for each game, so it may happen that the same question appears twice in a row. Let us now have a closer look at the type of questions in the game.

In the first kind of exercise, it is simply a matter of recognising true and false statements—that is, paying attention to the character making them. At each turn, you need to press either A or B to continue (if you do not press anything, you will not move on to the next turn). In particular, we will not follow the knave's clues but will instead do the opposite of what they suggest: the most complicated case is when

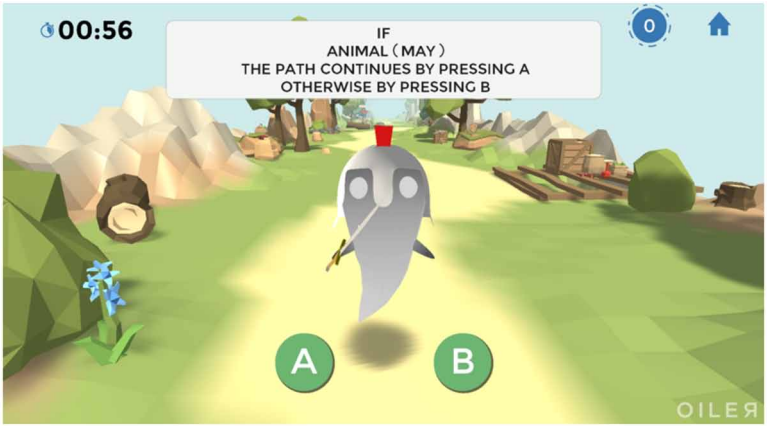
a double negation appears (from which we can see that we are in a classical logic-type environment). If the knave says "the path continues by not pressing A", we will have to press A regardless (see Figure 2).

Figure 2. The right answer is to press A



In the predicates section, phrases such as ANIMAL(tiger) appear: this notation should be read as outlined above. For instance, in figure 3 we see a knight with the speech bubble "If ANIMAL(May) the path continues by pressing A, otherwise by pressing B". Since May is not an animal, and the knight says the truth, we need to press B to continue.

Figure 3. Since May is not an animal, B is the right answer



The negation section introduces the *not* symbol (*i.e.* ¬). ¬COLOR(apple) should be read, as discussed, as "an apple is not a color". For instance, in figure 4 we see a knight with the speech bubble "If ¬ MONTH (London) the path continues by pressing A, otherwise by pressing B". Since London is not a month, and the knight says the truth, we need to press A to continue.

Il resto dell'articolo è disponibile su:

Bernardi, L. (2022). Bul Game: Playing With Knights and Knaves. In C. A. Huertas-Abril, E. Fernández-Ahumada, & N. Adamuz-Povedano (Eds.), *Handbook of Research on International Approaches and Practices for Gamifying Mathematics* (pp. 170–188). IGI Global. https://doi.org/10.4018/978-1-7998-9660-9.ch009