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Cognitive Skill Development: From Early Education to Labor Market Dynamics

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Chapter 1

Introduction

The realm of cognitive skill development, particularly in the context of economic outcomes, presents a multifaceted landscape for exploration and analysis. This thesis delves into the critical theme of cognitive skill development, tracing its trajectory from early education stages through its implications in labor market dynamics and in other relevant social domains characterized by uncertainty and ambiguity. Across three distinct yet interconnected articles, this body of work endeavors to shed light on the profound impact of cognitive abilities, their nurture, and their application in varied societal contexts.

The first article represents a distinct approach within this thesis, structured akin to a didactical mathematics paper rather than conventional economics research. It embarks on the foundational journey of cognitive skill development by focusing on the introduction of logical reasoning in primary education. This initial foray underscores the pivotal role of logic and systematic thinking in the formative years, setting a strong foundation for advanced cognitive abilities. The empirical evidence presented demonstrates the efficacy of a structured educational program in enhancing both cognitive abilities and mathematical literacy, thereby validating the hypothesis that early exposure to logic and reasoning significantly bolsters cognitive development.

Progressing to the second article, the focus shifts to the labor market, a domain where cognitive skills, once honed, manifest their utility and adaptability. Through a theoretical job search model, this study illustrates how an educational system that emphasizes cognitive skill training transforms initial abilities into a more versatile skill set. By analyzing data from the Programme for the International Assessment of Adult Competencies (PIAAC), the model highlights how changes in the educational system influence labor market dynamics, emphasizing the increasing importance of cognitive skills in a rapidly evolving job market. The findings suggest that an education emphasizing cognitive skills leads to enhanced labor market outcomes, including increased mobility and adaptability, reduced job polarization, albeit with the trade-off of lower initial productivity.

The third article extends the investigation of cognitive skills to broader social domains characterized by risk and ambiguity. Employing experimental and empirical methodologies, this study explores the relationship between intuitive cognitive abilities, associated with System 1 processing, and decision-making under uncertain conditions. The research underscores how individuals with higher intuitive cognitive abilities exhibit a greater tolerance for risk and ambiguity. Significantly, the study reveals that targeted training, such as the WarmApp activity (a novel computer-based tool designed to assess various domains of intuitive cognition), can effectively reduce ambiguity aversion, particularly in individuals with lower innate System 1 proficiency.

The variety of methodologies employed across these articles — from theoretical modeling to experimental and empirical approaches — demonstrates the multifaceted nature of cognitive skill development research. Each article, with its focus and methodology, contributes to a comprehensive understanding of cognitive skills' role in various life stages and societal contexts.

In conclusion, this thesis aims to advance our understanding of cognitive skill development and its application but also highlights the vital importance of these skills in navigating the complexities of the modern world. The collective insights from these studies offer valuable implications for educational policies, labor market strategies, and broader societal decision-making processes under uncertainty and ambiguity.

Chapter 2

Knight or knave? Description and evaluation of a programme for the introduction of logic at primary school

Riccardo Manghi & Luigi Bernardi
[1]

Abstract

The purpose of this article is to present and analyse a primary school educational programme based on the study of logic. The programme introduces logic through three main channels: linguistic reasoning, awareness of mistakes, and symbols. It supports the existing literature advocating the introduction of the study of logic in the early stages of education by presenting empirical evidence on how the educational path proposed increased both cognitive abilities and mathematical literacy in second-grade primary school classes.

Keywords Education, Logic, Math, Cognitive Skills.

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2.1 Introduction

Logic lies at the heart of mathematical and scientific thinking, and is fundamentally linked to certain elements of language. According to Ferrari and Gerla (2015), the constant attention paid to mathematical language, to the distinction between language and metalanguage, and to the notion of interpretation when working with logic makes it a tool suitable for teaching and learning at every educational stage. However, as pointed out in Durand-Guerrier et al. (2012) the educational role of logic is not always recognised. There may be several reasons for this: on the one hand, formal logic can be seen as an unnecessary tool that risks complicating teaching practice; on the other, some believe that basic logical abilities are developed irrespective of a targeted theoretical treatment. For example, the concept of 'not' is "considered as a very simple notion [. . .] that does not need to be taught or discussed at this [primary school] level" Durand-Guerrier (2021). But, according to the Author, the lack of an explicit treatment creates difficulties in understanding negation that can persist until university level, such as the fact that the connection between the negation of a universal statement and the role of counterexamples is never fully clarified. Another reason for the omission of logic in the school curriculum is that teaching of mathematics tends to prioritise exercises that develop the skills required for improving grades and meeting school targets; hence, any topics that are not deemed essential for future study — even if considered important for individual development — are left to one side.

The mistrust towards logic is majorly highlighted when dealing with symbolism. Returning to Durand-Guerrier (2021), neglecting the links between logic and language leads to the paradox that mathematical formalism, which should serve to clarify concepts, becomes an obstacle to students' learning. Indeed, logical formalism is often only seen when it is needed to express a mathematical concept not related to logic, and is viewed more as a syntactic abbreviation than a semantic clarifier. This is illustrated by the fact that a student may encounter quantifiers for the first time in the limit formula — a formula featuring three quantifiers as well as an implication — simply because it is no longer possible to express the concept in words. Introducing formal symbols for quantifiers this late in the game feels like a missed opportunity, akin to introducing the equality symbol for the first time when dealing with equations.

Introducing a symbol to denote a concept requires a societal agreement on its meaning, and allows us to become aware that symbols are related to the context of use Ferrari (2002). For instance, the logical conjunction AND will not capture every 'and' used in natural language; however, knowing how to recognise the differences and similarities in each case and context is an excellent starting point for learning the conjunction itself. Coppola et al. (2019)

provide an interesting analysis of the relationship between language, as an object to manipulate and reflect upon, and the development of logical abilities, considering specific scenarios of social interaction among primary school children (8–9 years old). A child is asked to behave like a robot that only obeys certain commands; the game therefore encourages the children to construct a simple symbolic language in which each symbol represents an instruction for the robot. These symbols do not correspond to those of standard logic, but the key point is to view logic as an "explicit expression of some aspects regarding language" Coppola et al. (2019). Moreover, the children can discover rules to "manipulate" the symbols of the created language (for example, rules that allow them to establish whether two different sequences of symbols can be considered equivalent in some way).

In this spirit, we present a programme for teaching logic at primary school. We tested the programme in two primary schools: two second-grade classes of an Italian school attended by many foreign students, and a fourth-grade class of a French school.

We believe that logic, even in its formal and symbolic form, supports the development of rational thought and that it is therefore appropriate to dedicate time and space to logic and its symbols from primary education onwards. In our opinion, the idea that an understanding of basic logical concepts can be acquired automatically through standard mathematical teaching is wishful thinking.

Anyways, we agree with some of the criticisms levelled at ways of teaching logic in which syntax is favoured over semantics: focusing solely on logical formalism, could risk stunting the affect and idea of mathematics. In our programme, every concept presented to the class is first introduced through discussions on language before moving onto the notation and formalism. As part of the programme, we introduce formal symbols to identify certain logical concepts, such as predicates and negation. These are simple concepts, much like equality and addition, for which the necessity of symbolic representations from the first years of primary education is universally recognised. Just as the equality symbol supports the development of language and algorithmic thought, we consider the negation symbol to play an analogous role in the development of language and rational thought.

Our hypothesis is that the explicit treatment of logic in primary education, in a game-like setting that integrates a variety of learning artifacts and dramatic approaches, will not only improve students' logico-mathematical ability (mathematical literacy, as defined in OECD PISA, 2021), but will also favour the development of cognitive abilities. To test our hypothesis, we formally analysed the effects of the teaching programme in one of the two primary schools mentioned above. We show how the programme appears to develop cognitive abilities related to so-called “fluid intelligence”. The description of the trial run is based on field notes and recordings from both primary schools, and some of the key educational moments of the programme are explored.

2.2 Conceptual Framework

As previously mentioned, our programme involves a playful approach to learning using a range of tools and techniques: theatrical activity, simulation of so-called Boolean circuits, discussions about symbols, worksheets on predicates, solving equations by trial and error, and the online game Bernardi (2022). Two characters are present in all of these activities: the knight and the knave. The island of knights and knaves is a well-known tale by Raymond Smullyan, describing an island whose inhabitants are either knights, who always tell the truth, or knaves, who always lie.

This island is mostly known from logical puzzles in which one is tasked with identifying the inhabitants of the island on the basis of their statements, with such puzzles making an appearance as early as primary education Carotenuto et al. (2017). Nonetheless, a story built around characters who speak the truth and characters who lie is likely to have a wider didactical value, not just because ”putting these matters in human terms has an enormous psychological appeal” Smullyan (1987), but also because — owing to the use and acceptance of falsehoods (those uttered by the knaves) — it offers a more playful approach to errors, which become part of the learning process rather than being immediately corrected.

One of the great questions in mathematical teaching is how best to approach and deal with a mistake made by a student. The teacher needs to juggle two elements: on the one hand, explaining to the student where and why the mistake was made, to avoid the error being repeated; and on the other, ensuring that the experience does not negatively affect the student’s relationship with mathematics. False statements are often firmly avoided in education: as maintained by Zan and Martino (2017) in an approach commonly found in mathematical teaching that is focused on training — and thus focused on reproducible processes — error is synonymous with failure and thus to be avoided. However, avoidance of error limits the possibility of dialogic learning, based on the comparison between truth and falsehood. Recent studies have looked at using incorrectly completed exercises for the purpose of student-conducted error analysis Rushton (2018). Such studies have shown an increase in mathematical understanding when working with a combination of correctly and erroneously completed exercises.

Errors can be divided into two broad categories: structural errors (i.e., syntactic), where symbols are used incorrectly (e.g., 3 + + + = =), and errors in meaning (i.e., semantic), where symbols are used correctly but the meaning is incorrect (e.g., $3 + 3 = 5$). A statement written with incorrect syntax does not have a truth value (it is neither true nor false), whereas a syntactically correct statement can be true or false. In our programme, the terms 'true' and 'false' are preferable to 'right' and 'wrong' when speaking about semantic errors: 'true' and 'false' have a clear logical meaning, and their use helps to place the two outcomes on equal footing. Furthermore, we prefer discussions on truth and falsehood to be done in the context of role play, where truth is represented by knights and falsehood is represented by knaves. Knights and knaves allow us to discuss semantic errors with ease, given that the errors made by knaves in their statements are always semantic, not syntactic. We believe that false statements can be used to improve understanding of symbols as well as concepts: knowing that $3 < 2$ is false helps to clarify the meaning of the symbol $<$.

In the classroom, students are rarely allowed to roam freely through the world of mathematics. Such roaming entails trying and failing (the word 'error' is derived from the Latin word errare, to wander or stray), and then modifying their approach on the basis of the information acquired. In our programme, anyone who makes a mistake is a knave — a character with whom students generally sympathise — and can make several mistakes before reaching a true statement and thus "becoming a knight". When exploring primary school teachers' opinions on logic, Bibby (2002) found the majority believe "the objectivity of logic contrasts with the apparent subjectivity of the creative process", viewing logic as an obstacle to mathematical discovery. This belief seems to be based on a limited view of logic, in which logic is reduced to a syntactic formalism without semantic value and, above all, is considered a technique solely related to deduction, "assumed as an unproblematic foundation for the justification of knowledge" Ernest (n.d.). We believe that logic has a broader scope and can aid in the art of discovery.

Our programme focuses on the study of syntax and its relationship with semantics. Through their island, the knights and knaves can help to define and delimit the context of the analysis and work to be done: the symbols we introduce can only be used on their island — in other words, within a logical and symbolic context — and not, for example, in an essay. First, we focus on the syntactic aspects of language and how they come together to create meaning. To do this, we look both at the words that make up our language and the structure that supports it — i.e. the rules that allow us to move from words to syntactically correct sentences. Through symbolism, we can highlight the role of structure with respect to words.

The module not only enables a different approach to error, but also permits an analysis of sentence structure. We believe that this methodology can explain why the tests we carried out seem to show an increase in cognitive abilities and mathematical literacy. We argue that continually translating between the structure of a statement and its interpretation can favour "proceptual thinking" which, according to Gray and Tall (1994), is a key determinant of a "successful thinker" when it comes to development of cognitive and mathematical abilities. Indeed, the ambiguity of notation — i.e., the role of a symbol both as a process and as a concept — "allows the successful thinker the flexibility in thought to move between the process to carry out a mathematical task and the concept to be mentally manipulated as part of a wider mental scheme". Gray and Tall also believe that "mathematical symbolism is a major source of both success and distress in mathematics learning", and that a successful thinker is able to "employ the simple device of using the same notation to represent both a process and the product of that process". We believe that our activity on truth and falsehood and on sentence structure works towards the "ambiguity" (as per Gray and Tall) of meaning, even if the logical symbols we use do not correspond to the operational symbols used in Gray and Tall's examples.

The importance of developing logically correct mental models at the primary-school level is highlighted in Kuhn et al. (2000). In this paper the authors argue that inquiry-based learning "at and from" middle-school level can be compromised by students having flawed mental models of causality. The paper shows how the simultaneous occurrence of a given value of a variable in a multivariable system and a particular outcome can be sufficient for students to expect a causal relationship between the variable and the outcome (in particular, the students struggle to conceive the outcome as being independent from the variable, and thus unaffected by the latter). The authors call this flawed model the co-occurrence model. This problem is not only due to a misunderstanding of causality, but also a failure to account for the additivity of the individual factors (i.e., their combined contribution) within a multivariate system. We believe that a true understanding of causality and additivity can only be reached after previous study of logical connectives. On the one hand, logical implication — which has no causal value — illustrates how co-occurrence is not sufficient for causality; on the other, the use of the connectives AND and OR with independent variables helps to develop the mental model required to "deconstruct" the total effect into that of the individual factors, providing the background required to understand additivity. Although the activity proposed here does not cover logical implication and only briefly touches upon the connectives AND and OR, it nonetheless lays the necessary groundwork to address those topics. In fact, the complete programme, which is yet to be tested in the classroom, features a more explicit introduction of AND and OR, and the game Bul Game provides a first encounter with implication and its negation, referred to in the game as exclusion (borrowing the notation from set theory).

Therefore, the study of logic in primary education — including the experiences proposed here — aims to prevent the development of the flawed mental models described in Kuhn, which impede examination of multivariate systems; in doing so, they prevent the possibility of carrying out inquiry-based learning (IBSE) in the higher educational phases, which is increasingly supported by the main European educational institutions "PISA 2021 Mathematics Framework Draft" (2021).

2.3 Trial Run

The activities presented here have been used in two second-grade classes (ages 7–8 years) of an Italian primary school and in a CM1 class (ages 9–10 years) of a French primary school, to introduce students to propositional logic "Bul game" (2021).

2.3.1 Description of the Activities

In the following descriptions, we refer to both the researcher and the class teachers who assisted as teachers.

Phase 1 Italian School: Theatrical Activity

The aim of the first activity was to introduce students to the knight and knave characters. Masks for the two character types were prepared in advance to be given to each student when needed. The knight character was introduced first, described as someone who always tells the truth. After being given an example by the teacher, the students were then encouraged to make true statements on any topic while wearing a knight mask. The students immediately grasped the concept of a true statement, providing statements about their immediate surroundings, and then moved onto general truths. Some statements were not verifiable (e.g., "my sister is called Maria"), but they all had a truth value. No students suggested phrases that did not correspond to a statement, such as "the umbrella", nor phrases without a truth value, such as "it will rain tomorrow". The students were then encouraged to come up with mathematical statements once the teacher had provided an initial example of a simple identity such as "2 plus 2 is equal to 4". The knave character was then introduced as someone who always lies, and — as with the knight — the students were encouraged to make false statements, first generally and then within a mathematical context. Although the teacher's initial examples were of incorrect calculations, such as "2 plus 3 is equal to 2", some students opted for diverse examples of mathematical falsehoods, such as "100 has two figures".

The next part of the activity was more similar to Smullyan's classic puzzles, where students had to work out whether the character speaking was a knight or a knave. This activity required two teachers, one who wore a knight or knave mask with their back turned to the class and provided riddles for the students, and another who helped the students to solve the riddles and identify which character was speaking. To start with, the riddles were very simple, as they did not follow the classic formulation seen in Smullyan's puzzles (which self-refer to the same group of characters speaking) but were simple statements such as "tigers can fly". The students were then asked in turn to play the role of the knight or knave and provide riddles for their classmates. The teacher then introduced the emblematic statement "I am a knight", always with their back turned to the class. After initial attempts to reach a decisive solution — during which both characters were suggested — the class realized that it was not possible to know whether the person speaking was a knight or a knave on the basis of that statement alone. This provides the students with an example of a question to which there is no single correct answer. Similarly, the class was encouraged to consider the phrase "I am a knave" and were pleased to discover that neither character would have been able to say this phrase. At the end of the first part of phase 1, some of Smullyan's simpler classic riddles were proposed to the class, which involved more than one masked character with their backs to the class (one teacher and one or more students). The teacher told each of the masked students what to say and the rest of the class was asked to deduce their identity. It is important to note that creating physical representations of the characters making these statements — with their backs turned and faces hidden, but nonetheless there in person — is likely to have made it easier for the students to solve the riddles.

To finish off the activity, the class was introduced to "Boolean circuits", using the knave as a representation of 'false' and the knight as a representation of 'true'. This choice works on a logical level, given that for every statement A made by a knave, we have $A \leftrightarrow$ FALSE, and for every statement B made by a knight, we have $B \leftrightarrow$ TRUE. The aim in each circuit (see Fig. 2.1) is to reach the final circle — shown in red in the figure — wearing a knight mask. At the start of the circuit (in the blue circle), the player chooses which mask to wear. The first circuit proposed is trivial, whereby the player simply has to follow the rope to the finish circle, with no unexpected events along the way.

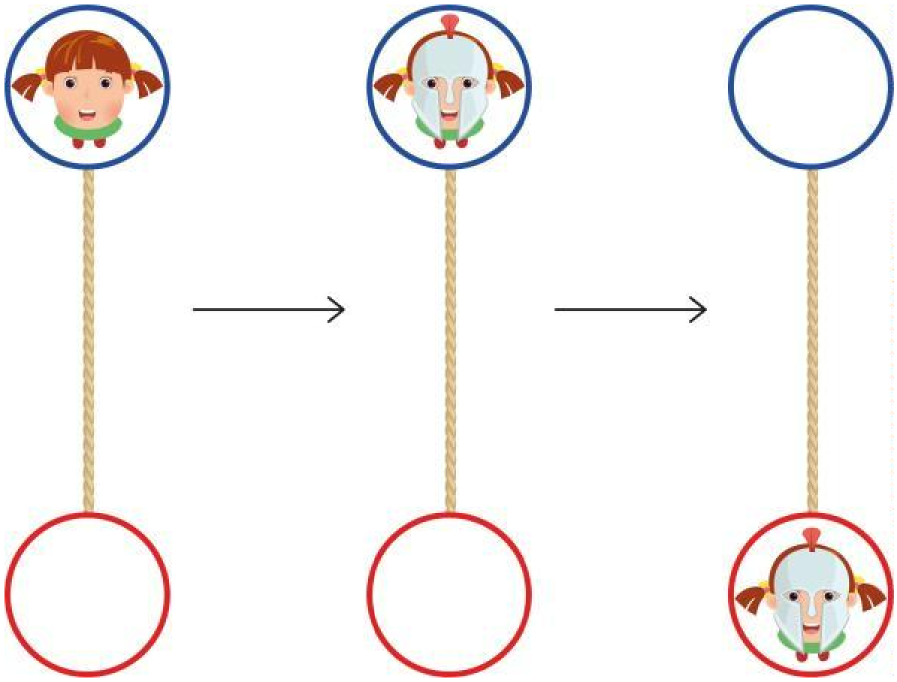

Figure 2.1: The first circuit.

The second circuit introduced Dr. No, a character (played by a student) who forces any player who encounters them to change their mask (Fig. 2.2). The winning strategy, as shown in figure 2.2, is to start the circuit wearing the knave mask.

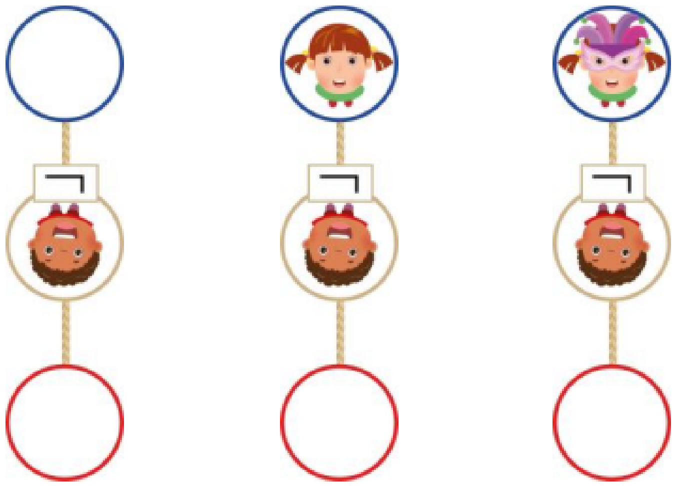

Figure 2.2: To solve the negation circuit, you should start the circuit with a knave mask.

The next circuit then included two Dr. No's (Fig. 2.3), one after the other; here, the winning strategy is to start the circuit wearing the knight mask. More Dr. No's were then introduced sequentially into the circuits, leading towards a discussion on the parity of the number of negations: if there is an even number of Dr. No's — including none at all — the winning strategy is to start with the knight mask; if there is an odd number of Dr. No's, the winning strategy is to start with the knave mask. After a few initial mistakes, all of the students understood the winning strategy and were able to choose the mask needed to successfully complete the circuit. The relationship between the winning strategy and the parity of the number of negations was highlighted.

Figure 2.3: Multiple negations circuit.

The final circuit introduced the connectives AND and OR. Dr. AND is a character who prefers knaves: if approached by a knight and a knave, Dr. AND will let the knave pass; if approached by two knaves, they will let the knave of their choosing pass; and if approached by two knights, they will be forced to let a knight pass. Dr. OR is a similar but opposite character to Dr. AND, instead preferring knights: if approached by a knight and a knave, Dr. OR will let the knight pass; if approached by two knights, they will let the knight of their choosing pass. This way of using AND and OR corresponds exactly to the truth tables of the two connectives, positioning each connective as a rule of deduction rather than a symbol with a particular meaning. The class was able to easily complete the circuits; however, no in-depth analysis was done of the various winning strategies.

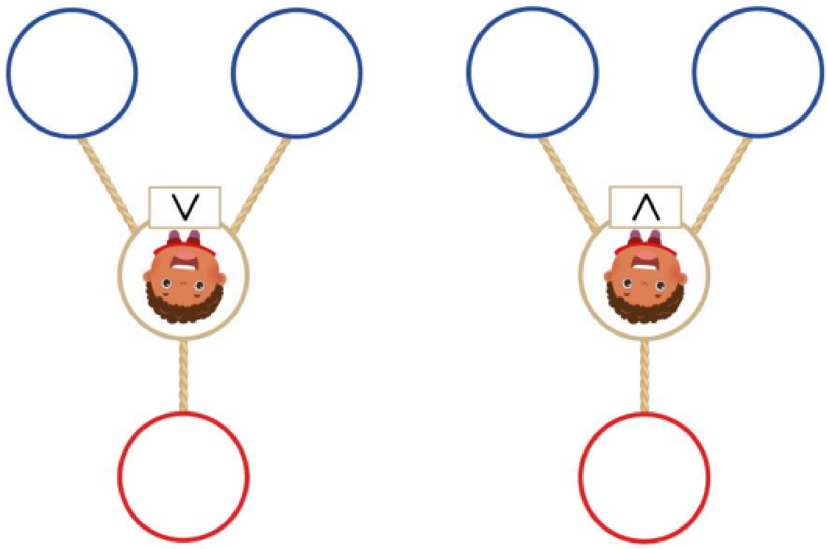

Figure 2.4: Circuits with AND and OR.

Phase 1 French School: Theatrical Activity

The activity with the fourth-grade class followed the same steps as with the second-grade class, but at a quicker pace. Notably, a student made a statement that triggered the discussion of a key point, declaring “we are all girls” while wearing a knave mask; this statement opened the floor to a discussion on the terms ‘all’, ‘at most’, ‘at least’, and ‘none’. When initially questioned, the students fell foul of the classic mistake of thinking the negation of ‘all’ to be ‘none’, and vice versa. To address this, the teacher laid a pink hula hoop on the floor and said “all of the circles are pink”; the class correctly identified the teacher as a knight. The teacher then put another pink hoop on the floor and repeated the phrase. Again, the class correctly identified them as a knight. The teacher added yet another pink hoop to the collection, and the discussion was repeated. Finally, the teacher laid down a green hoop: at this point, there were three pink hoops and one green hoop on the floor. The teacher then said “all of the circles are pink”, at which point the class correctly identified them as being a knave. In this way, the class was able to explore the key concept by which the negation of ‘all’ is ‘there exists one that is not’. Knights and knaves are a useful tool for the analysis of such words: negation of quantifiers can be hard to grasp, but identifying with a character who lies or speaks the truth can aid in understanding. Throughout this conversation the students began to discuss elements of set theory, even if it is not explicitly part of this programme. This first phases in both schools lasted around 1 hour and 30 minutes. To consolidate the concepts learned in this phase, and to approach them from another perspective, the classes the classes played Bul Game before moving onto phase 2.Bul Game is an online game in which players encounter either a knight or a knave in each turn, and progress through the game by choosing between two options on the basis of a statement given by the knight or knave Bernardi (2022). The types of riddles encountered are classified into sections that reflect the stages of the activities. Before phase 2, all classes played the first section of Bul Game, TRUE&FALSE.

Phase 2 Italian School: predicates

The students were told that knights and knaves sometimes communicate with one another using a strange way of writing. First of all, students were asked to pick out the key elements of a phrase such as "a tiger is an animal", identifying 'tiger' and 'animal' as essential words to understand its meaning. More accurately, the central components of the phrase are the predicate "being an animal" and the object (in this case, the subject of the phrase) that the predicate refers to. Anticipating the next step, a student noted that "a 'not' would be important too if it were there". The students were then given parentheses to colour in, to familiarise them with the symbol. The students were then told that knights and knaves use the two words 'tiger' and 'animal' and parentheses to write the phrase "a tiger is an animal". Some students suggested TIGER(ANIMAL) as a potential solution, and others (TIGER ANIMAL) (which is somewhat reminiscent of Barandrecht's lambda calculus!); a few other students suggested ANIMAL(TIGER). Each of these three notations can be used without leading to contradictions. The students were finally told that knights and knaves use the notation ANIMAL(TIGER). This is the standard notation used in logic and general mathematics, where the object of the predicate, or function, sits within parentheses after the symbol for the function. We feel that this early introduction of formal notation can be beneficial: first, as mentioned in the introduction, it allows students to become accustomed to using a symbolic and context-dependent language (this language is used exclusively on Smullyan's island of the knights and knaves). This situation highlights the fact that changing language does not necessarily involve changing the vocabulary or alphabet; the formal language outlined here shares the same words and symbols as English, but applies them using different rules. The key point is to create a broader view of language, which is not defined exclusively by its alphabet and vocabulary but also by the rules that govern the construction of phrases Bernardi (2022). Furthermore, by considering a range of objects that either verify or falsify a given predicate, one is gradually able to identify and isolate the specific characteristics — i.e., the properties — that characterise objects that satisfy that predicate.

In other words, notation such as ANIMAL( ) encourages the transition from an extensive description (based on many examples) to an intensive description of being an animal. Finally, the formal structure of predicates makes it easier to write phrases with the negation symbol, as we will see in the following phases. Importance was placed on the translation from symbolic form to natural language: TREE(OAK) should not be read “tree oak” but “an oak is a tree”. The students were given statements to translate in both directions, with examples of true statements — i.e., those made by a knight — such as ANIMAL(TIGER), and false statements — i.e., those made by a knave — such as ANIMAL(TABLE). To finish the activity, students were given exercises in which they were asked to correctly complete predicates according to which character was speaking—for example, EVEN(. . .) or 3 < . . . (Fig. 2.5). It should be noted that the students were free to fill in the predicates as they pleased. If a knight is speaking and we write CITY(x), x is necessarily a city. But if a knave is speaking, x can be anything that is not a city. Nonetheless, most students favoured the more meaningful contexts: NOT CITY(FRANCE) makes more sense than NOT CITY(9). Notably, with the predicates EVEN( ) and ODD( ), all students eventually opted for numerical contexts.

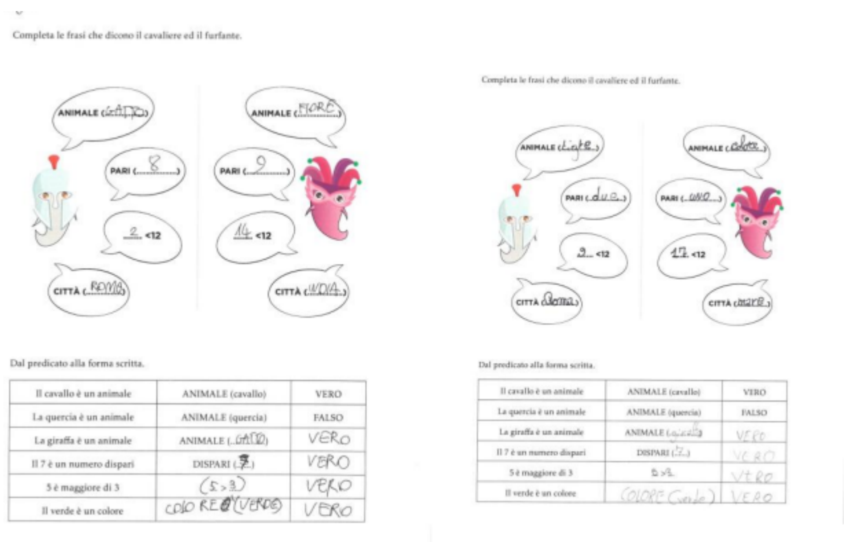

Figure 2.5: Exercises done by students.

Phase 3 Italian School: negation

The phrase “a tiger is not an animal” was written on the board and the students were asked, as in phase 2, to identify the key words. It was noted that, in addition to ‘tiger’ and ‘animal’, the word ‘not’ was also fundamental. A few exercises were done on the board whereby students needed to work out whether a given phrase had been said by a knight or a knave; for example, the first phrase was said by a knave, whereas the phrase "3 is not even" was said by a knight. The class was then given the negation symbol $\neg$ to colour in, to familiarise them with the symbol. Many recognised the symbol from the first phase, when it was used with the Dr. No character. Following this, the students carried out translation exercises — first orally at the board, and then written — and were given comics to fill in, depending on whether the person speaking in the comic was a knight or a knave. The phrase "red is not a colour" would be translated as $\neg$ COLOUR(RED). Similarly, the phrase $\neg$ ODD(4) is translated as "4 is not an odd number". We highlight here that two different approaches were taken for the negation symbol. In phase 1, the symbol was introduced as a rule: the symbol acted on the truth value of a statement by changing it — that is, by changing the mask worn. In phase 3, the negation symbol was introduced as a logical connective with semantic value. These two interpretations are clearly very closely connected. If either a knight or a knave writes the phrase PREDICATE(OBJECT), then the introduction of the negation symbol will force a character swap, because the phrase $\neg$ PREDICATE(OBJECT) can only be written by the other character. Until this point, the statements provided that contained the negation symbol had been limited to the form $\neg$ PREDICATE(OBJECT). The statement $\neg (3 \leq 2)$ was then written on the board and a student was asked to translate it. Surprisingly, the student translated it as "3 is not less than 2", applying the negation to the predicate. Exercises have been proposed to review the negation (Fig. 2.6). It is worth noting that in natural language, the position of a negation is often not well defined a priori; in some cases, moving its position does not affect the meaning of the phrase (e.g., "all people do not have blonde hair" is equivalent to saying "all people have non-blonde hair"), whereas in other cases, moving the negation can distort the meaning (e.g., "not all people have blonde hair" is completely different from "all people do not have blonde hair"). Further examples relate to double negations, which work as affirmations in some languages and negations in others. We believe that introducing the negation symbol and its rules aids in the understanding and clarification of these various situations.

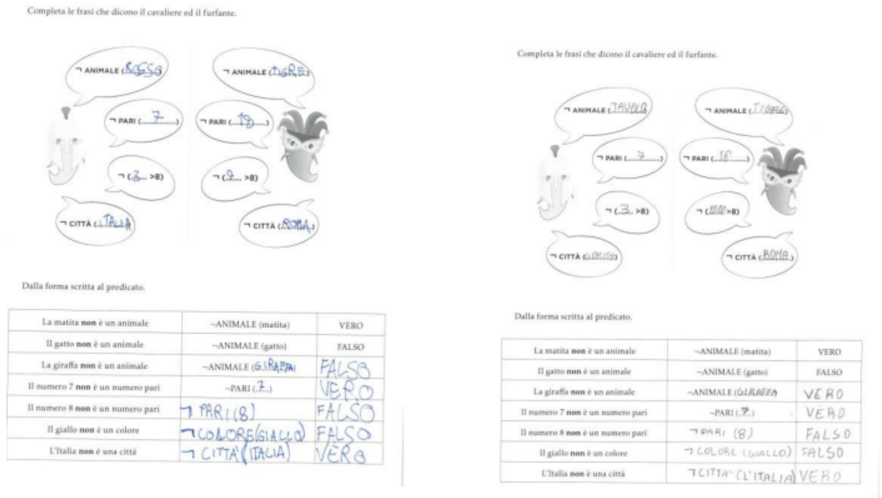

Figure 2.6: Exercises done by students.

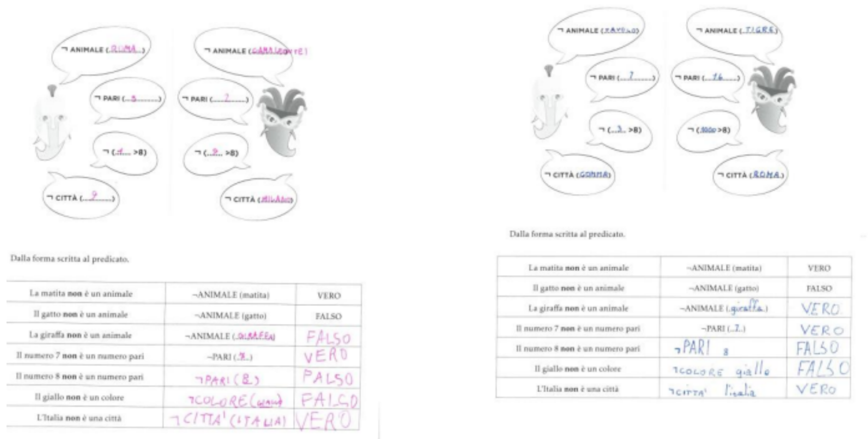

Figure 2.7: Other exercises done by students.

Phase 2 French School: predicates and negation

For the fourth-grade class, phases 2 and 3 of the activities were merged (Fig. 2.7). Furthermore, binary predicates involving two objects were introduced. The first binary predicate to be introduced was MOTHER(x, y), with the teacher going through several examples with the students; the chosen convention was that x is the mother of y. Examples of this predicate were given where a knight was speaking, as well as where a knave was speaking. The predicate FRIENDS(x, y) was then introduced, with further examples. It was noted that writing MOTHER(x, y) is different to writing MOTHER(y, x) (in fact, one case precludes the other), whereas writing FRIENDS(x, y) is equivalent to writing FRIENDS(y, x). This property of the latter was described to the class as symmetry, which is common in mathematics: for example, the binary predicate < is not symmetrical whereas the binary predicate = is. Finally, the similarities between symmetry and commutativity were highlighted to the class.


Un peu d'exercices !

ANIMAL (tigre)	Le tigre est un ANIMAL	VRAI
COULEUR (chaise)	La chaise est une COULEUR	FAUX
ARBRE (chêne)	Le chêne est un arbre	Vrai
ANIMAL (table)	La table est un animal	Faux
VILLE (Rome)	Rome est une ville	Vrai
FRUIT (banane)	La banane est un fruit	Vrai
PAIR (7)	Le 7 est un nombre pair	Faux

Le crapaud est un animal	ANIMAL (crapaud)	VRAI
Le vase est un animal	ANIMAL (vase)	FAUX
Le sapin est un arbre	ARBRE (..sapin)	Vrai
Le fer est un métal	MÉTAL (fer)	Vrai
5 est un nombre impair	IMPAIR (5)	Faux
Le soleil est une planète	PLANETE (soleil)	Faux

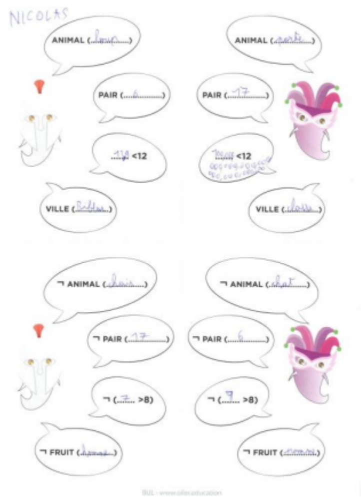

Figure 2.8: Exercises done by students.

Figure 2.9: Other exercises done by students.

Phase 4 Italian School: Variables (Gym Hall)

This part of the programme concerns the search for a solution via trial and error (analysis in the sense of analysis-synthesis environment). We note that a univariable equation is a particular type of unary predicate. Use of the knave character seems to help children through the analysis stage by removing the fear of making mistakes.

The gym hall was marked with large sheets of paper containing an x on the floor. The aim was to complete the equations, inequalities, or predicates—such as `EVEN(x)`—by placing an appropriate number over the x; in other words, substituting a constant for a variable.

To make students comfortable with the notation, the variable x was firstly introduced as a mystery number. Questions such as "I know a number x such that $x + 3 = 8$. What number is it?" or "I know a number x which, when added to itself, makes 10. What number is it?" were posed. It was not hard

Figure 2.10: Playing with equations.

for the students to answer these first simple questions. Such equations, which are generally first encountered in middle school (ages 11–14 years), are usually solved using a synthesis method that relies on inverse operations. We clearly did not consider it appropriate to introduce such a method at primary school and the equations were instead solved by trial and error: different numbers were substituted for x on the board and the resulting equality was checked.

More complex statements were proposed to the class, such as `EVEN(x)`, and the class noted that, this time, there were many different possible solutions. Multiple requirements were therefore added together: "I know a number x such that `EVEN(x)`, $x < 10$, and x is a three-letter word. What number is it?" In this case, there is still more than one solution, but the number of solutions is finite.

The children were allowed to work freely, and enjoyed coming up with numbers they wanted to substitute for x, trying out a wide range of numbers (a self-critique: it would have been better to present a wider range of predicates to the class). By that time, the class was used to recognising false statements (and judging them as such) thanks to their familiarity with the knave character. We believe that solving through trial and error should also be encouraged in older year groups to make students more comfortable with errors and falsehood. Once an equation was solved by a student, it was put aside.

This trial-and-error approach is an excellent example of roaming freely through mathematics.

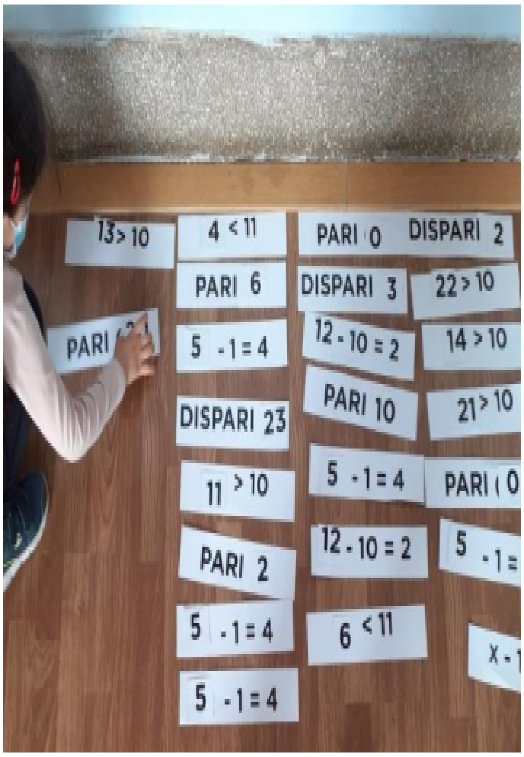

Figure 2.11: Putting (not) all correct predicates aside

The activity described above offers the perfect opportunity to discuss proceptual thinking, as outlined in the theoretical framework. Children of primary-school age are not yet able to manipulate equations (i.e., moving elements from one side to the other) and thus cannot carry out the “process” represented by the equation: an equation — before any substitutions — is simply a concept. Once a constant (number) has been substituted into the equation, the equation is processed. If the equality obtained is false, you go back to the original concept, more knowledgeable than before.

Phase 3 French School: variables

The terminology already discussed in phase 1, such as ‘all’, ‘at least one’, and ‘none’, was reviewed through examples, as was the concept that the negation of ‘all’ is ‘at least one is not’ and the negation of ‘none’ is ‘at least one is’ — all with the help of knights and knaves. The concept of a variable was then introduced, using similar examples to those used in the second-grade class. This concept was then incorporated into the previous activity. A formula was written on the board that contained x and the students were asked whether the elements x that satisfied the formula (referring implicitly to natural numbers) were all, some (i.e., at least one; the students did not seem to have a problem with it being exactly one), or none. For example, $x + 3 = 5$ is satisfied by one number, whereas $x = x$ is satisfied by all numbers; by contrast, $x > x$ and $x + 3 = 1$ are not satisfied by any natural number. The same question was posed about the predicates EVEN(x) and ODD(x), noticing that, even if not all numbers satisfied the predicates, both were satisfied by infinitely many numbers. Furthermore, it was pointed out that EVEN(x + x) is satisfied by all natural numbers. The class was asked to find an equivalent expression such that ODD(expression) was true for all natural numbers. At first, the class had no idea how to approach this problem, but then began to work out what sort of expression would be required. They were placed into small groups to work on a solution, supported by three teachers. The students suggested solutions such as ODD(x − x + 1): they were told that, while correct, these expressions always give the same result, regardless of the value of x, and were encouraged to find a non-constant expression. After a while, several students independently concluded that a possible solution was ODD(x + x + 1). The rest of the activity followed the same steps as for the second-grade class, with numbers laid out on the floor, alongside formulas to be completed, with a few more complex examples introduced.

2.4 Empirical Analysis

In this section, we explain how we assessed the causal impact of the BUL educational programme in the development of cognitive skills and mathematical literacy, in order to provide empirical evidence supporting our main argument. The analysis is structured as a randomised controlled trial (RCT).

2.4.1 Methodology

The empirical exercise was structured as follows: we selected two different second-grade classes in the Italian primary school. One class was the intervention group, in which the educational programme BUL was run, and the other class served as the control group, receiving the usual programme of teaching. To avoid potential sources of endogeneity, the control class was in the same year as the intervention class, and shared the same teacher. We validated the RCT assumption of randomisation by using the proper balancing test to check that class compositions were as good as random with respect to relevant covariates (age, sex, and nationality of origin), and then measured mathematical literacy and cognitive skill in both classes, before and after the intervention, and calculated the score difference.

We measured mathematical literacy using INVALSI questions. INVALSI are national tests specially designated and recognised by the Italian state to evaluate skills (understood as knowledge and ability to think about knowledge) in fundamental areas such as mathematics, Italian, and English. Questions from the mathematics INVALSI tests therefore provide a good measure of mathematical literacy. The pre-intervention and post-intervention tests used in this trial were composed of different sets of four past INVALSI questions. An example of one of the INVALSI questions used is shown below (Fig. 2.12):

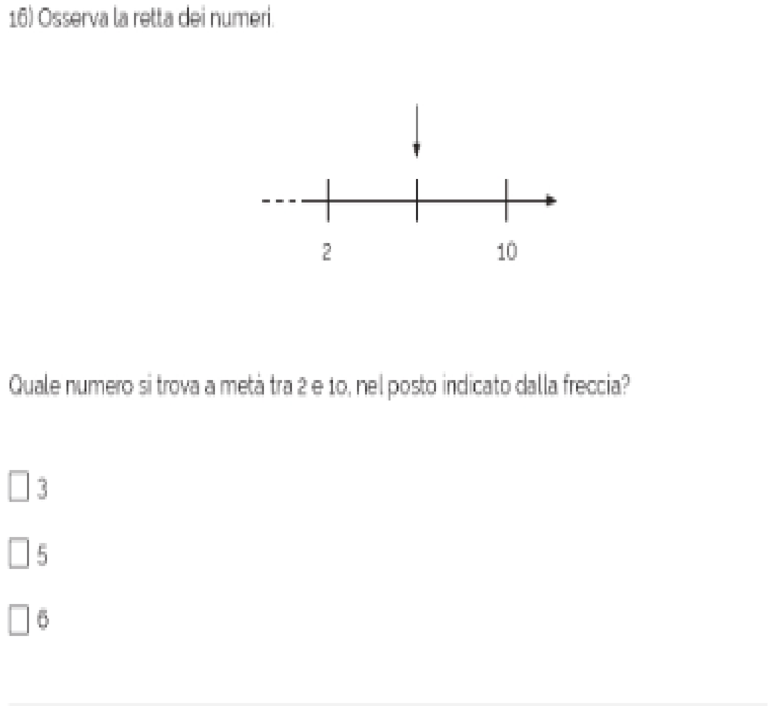

Figure 2.12: In the above question the student is asked to say which number lies between 2 and 10.

We measured cognitive skills with Raven's progressive matrices. This nonverbal test is widely recognised as a measure of fluid intelligence, which refers to the ability to solve novel reasoning problems, and is correlated with several important skills such as comprehension, problem solving, and learning in individuals aged 5 years and older Kaplan and Saccuzzo (2009). We used 13 questions from the Colored Progressive Matrices (RCPM) variant, a version of the Raven test designed specifically for children aged 5–11 years Domini and Domino (2006). An example of one of the RCPM questions used is shown below (Fig. 2.13):

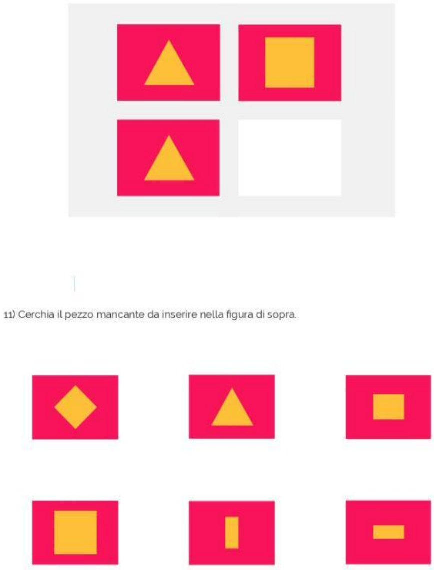

Figure 2.13: The student is asked to choose the missing piece

2.4.2 Data

We included all students from the intervention and control classes who completed both the pre-intervention and post-intervention tests. After excluding students who scored the maximum in the first test — and therefore could not show improvement — our final dataset was made up of 18 students in either class, whose characteristics are summarised in table 2.1:

	Control (N=18)	Intervention (N=18)
Mean RCPM score	10.3	10.2
Males (%)	9 (50%)	11 (61%)
Non-Italian origin (%)	5 (28%)	5 (28%)

Table 2.1: Characteristics of the study participants

Here, we have included all observable variables that may affect the rate of improvement in the tests used. As the students were all of the same age, we performed balancing tests for sex, nationality of origin, and initial RCPM score. Given the relationship between learning and intelligence outlined by Jensen

(1989), it is possible that initial cognitive ability can affect the rate of learning of both mathematical literacy and cognitive skills. Furthermore, according to Vaci et al. (2019), the benefits of practice increase with intelligence, suggesting that a child with higher initial cognitive skills would be able to improve their mathematical literacy more than their peers just from the standard math classes. We therefore included the initial RCPM score in the relevant characteristics.

2.4.3 Statistical Analysis

To assess the randomness of class compositions, we used a Student's t-test to compare the mean values of each covariate between the two groups; we chose this statistic because the variance of each covariate was similar between groups, and Student's t-test is appropriate for very small samples de Winter (2013). The results are showed below:

	RCPM score	Sex	Non-Italian origin
t statistic	–0.23	1.07	0
p-value	0.42	0.62	1

Table 2.2: Results of the balancing test

The results show no significant difference between the two groups, validating the assumption of the class compositions being as good as random. We therefore ran two unadjusted regressions with score difference as the independent variable and intervention group as the dependent dummy variable, for both mathematical literacy and cognitive skills. In this analysis, the regression coefficient of the intervention variable represents the mean difference in the outcome between the intervention and control groups. Since the sample was relatively small, we set the significance level at p=0.1.

An important issue that we were unable to adjust for is the potential influence of a memory effect. As the post-intervention RCPM test was composed of the same questions as the pre-intervention test, our results may be biased by this effect, even if the students were not given solutions to the tests.

2.4.4 Results

The results are shown below:

We observed a marginally significant effect ($p<0.1$), of the intervention on both cognitive skill and mathematical literacy. Notably, the regression coefficient for cognitive skills is smaller than that for mathematical literacy, indi-

	Cognitive skills	Mathematical literacy
Coefficient	0.55	2.27
Standard error	0.29	0.88
p-value	0.06	0.09

Table 2.3: Results of the regression analysis

cating a smaller difference in scores between the two groups, but the smaller p value indicates it approached acceptable levels of statistical significance (p=0.05). This result may be due to memory effect bias and the small sample size, as discussed above. However, there is no reason to suppose that memory effect bias is more prominent in one group than in the other, so this issue does not invalidate our findings.

2.5 Conclusion

The RCT performed showed that the educational programme proposed helps to develop mathematical literacy and general cognitive skills, specifically fluid intelligence, stimulating the formation of mental models that support the continued development of skills and abilities throughout the various stages of education. As argued in the theoretical framework, the improvement that takes place stems from a variety of sources: reflection, analysis of error, familiarity with logical symbolism, development of thought at the metalevel via in-depth study of the relationship between syntax and semantics, stimulation of proceptual thought, and learning about logical connectives and their use.

Our theories and empirical findings support the existing literature Durand-Guerrier (2021), Ferrari and Gerla (2015), Coppola et al. (2019), all of whom advocate for the introduction of the study of logic from primary education onwards, and argue that the study of language can help to develop logical abilities.

Our next steps will be to analyse the outcomes of another educational programme based on the study of set theory—named Zermelo—in order to further develop the topics addressed in this article, and to conduct a more detailed analysis on students' emotional response to these programmes (i.e., their engagement with and perception of mathematics).

2.6 Acknowledgements

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Chapter 3

How Cognitive Skill Development Impacts Job Market Outcomes: A Theoretical Analysis

[1]

Abstract

This article analyzes the impact of different learning methodologies on the development of skills and their consequences in the labor market through a theoretical job search model. The paper presents an equation for modeling the dynamic acquisition of skills during education and applies it to a job search model with on-the-job training. The model is calibrated using data from the Program for the International Assessment of Adult Competencies (PIAAC), and explores the optimal design of the educational system with the goal of maximizing the aggregate match value in the labor market. The results indicate that a shift towards a greater emphasis on cognitive skills leads to improved labor market outcomes, including increased flexibility and mobility, and reduced skill polarization.

Keywords Education, Job Search, Mismatch, Learning, Skill polarisation.

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3.1 Introduction

To what extent the ability to adapt skills to different requirements is becoming important in a world of rapid technological advancement and labor demand changes? How should the educational system adapt? Can the government increase workers' flexibility through cognitive skill training during education?

The ability to adapt skills has become increasingly important in today's rapidly changing world with technological advancements and shifting labor demands. The continuous evolution of technology and computerization has dramatically altered the labor market in recent decades. This has resulted in a significant decline in the demand for routine jobs, regardless of whether these jobs are cognitive or manual in nature, and a corresponding rise in the demand for both high-skilled and low-skilled workers. This shift has been referred to as ”job polarization” and has been extensively studied in academic literature, with a focus on the factors driving these changes and their impact on workers and wage inequality (Acemoglu and Autor (2011); Katz and Autor (1999); Commander and Kollo (2004), Machin (2003), McIntosh (2002)). The trend of job polarization is expected to persist, making it increasingly important for individuals to possess skills that are flexible and transferable to a variety of different roles and industries.

The impact of digitalization on the workforce is significant, affecting not just the volume and type of work, but also the way it is organized. An increasing number of tasks will be performed online and made tradable over the internet (Dachs (2018)). This means that companies and senior executives need to reevaluate their role in preparing workers for a rapidly changing economy by developing the necessary skills for new job demands (Illanes et al. (2018)).

As technology continues to evolve at a rapid pace, the ability to adapt to changing work and skills requirements is becoming increasingly important. This issue is addressed in the book ”Anticipating and Preparing for Emerging Skills and Jobs” (Panth and Maclean (2020)). The authors argue that there is an urgent need to align education and training with the current and future needs of the economy, as the gap between acquired skills and labor market demands will only continue to widen.

New job opportunities will be created as technological, social, demographic, and political changes continue to accelerate, creating volatility and uncertainty. A future-oriented curriculum framework is required to enable learners to prepare and adapt to these changing needs with confidence. This requires a clear and effective action plan to match education and training systems with the rapidly evolving employment needs of each country.

Expert consensus indicates that the current gap between formal skills and com-

petences and labor market needs will not only persist, but will widen in the future (Redecker et al. (2010)). To effectively address these changes, an overhaul of the educational system, which was designed decades ago, is necessary.
This research proposes a novel framework to examine internal efficiency of education by integrating job search models with more traditional models of human capital, through the presence of skill accumulation during education. The model seeks to bridge the divide between educational skill development and labor market dynamics, aiming to elucidate the mechanisms through which educational policies shape employment outcomes in labor markets. The basis framework I propose is a theoretical job search model that exhibits on-the-job training, along the lines of Postel-Vinay and Lise (2020), with two major additions: the rate of on-the-job skill adjustment is endogenous and depends on a particular component of the two-dimensional skill vector (cognitive skills), that is affected by the design of the educational system chosen by a theoretical planner. This setup requires to address two particular issues: identify which skill determines the rate of skill adjustment, and verify that this skill is actually trainable during education. Concerning the former, the literature in cognitive and learning psychology agrees that learning ability depends on cognitive skills. To cite Jensen (1989) <<review of evidence from psychological and educational literature on the relationship between individual differences in measures of learning and of intelligence suggests that no clear distinction can be made between the two cognitive processes>>.
Jensen concludes that both learning and intelligence reflect Spearman's g factor, which is thus related to the cognitive skill notion. Moreover,Gathercole et al. (2019) provides evidence that development of new routines depends on general cognitive resources (in particolar the so called "fluid intelligence").
The dependance between the rate of skill adjustment and cognitive skills will be validated and modeled using data from the italian subset of OECD PIAAC (Survey of Adults Skill), that will be also used to calibrate the whole model.
Regarding the trainability of cognitive skill, especially during the first stages of education, research in cognitive and learning psychology points to the fact that different methods of learning can lead to significant improvements in cognitive skills. For instance, inquiry-based learning has been found to lead to a better understanding and improved cognitive skills (Guo (2010); Ismail and Elias (2006)). Studies have also shown that cognitive skills can be improved through traditional methods such as memorization, practice, and problem-solving (Carpenter et al. (1990); Hegarty (2004)). In summary, cognitive skills are trainable and can be improved through various methods of learning.
In the first chapter of this thesis, I analyzed how the study of logic in primary school helps to develop skills not only in mathematics, but also more generally.

We provided empirical evidence designing an RCT that showed how an educational path based on an inquiry-based approach to the study of logic increased performance on a Raven matrix test for children. This RCT was registered in the American Economic Association of Randomized Control Trials, and is part of a larger research project involving other educational pathways characterized by an approach based on role playing and inquiry based learning in various areas of mathematics. The data collected in these projects supports the second assumption: that is, cognitive skills, and in particular the so-called "fluid intelligence", are trainable during the education. Therefore, changing programs and methodologies in the scholastic system, the government is able to strategically realign the initial distribution of multidimensional skills among individuals, placing a greater emphasis on cognitive abilities, at the expenses of practical tools and notions that will be needed to acquire marketable knowledge.
The search model is then used to describe formally the dynamics through which the development of different types of skills translates into a trade-off between flexibility and initial productivity, and to analyze the implications that a shift into flexibility has into wages' distribution and path, job mobility, value generated by the job market and capability of the economy to adapt to technological innovation and sectorial shifts.
First I propose an equation for skills dynamic during education. The functional form for such equation proposed is consistent with the literature, as in Sanders (2012), where the skill accumulation equation also depends on cognitive skill, but has a stochastic component. The functional form is also supported from the empirical literature in cognitive and learning psychology, in particular to Vaci et al. (2019) In this longitudinal study that tracked the evolution of chess' ability depending on intelligence and practice, three main results are presented: more intelligent people benefit more from practice, linear returns from practice, diminishing returns from intelligence. The dynamics of skill accumulation are thus modeled through a Cobb-Douglas function, which quantifies the gains in skill as a function of two key inputs: cognitive ability and practice. This formulation allows us to capture the multiplicative interaction between cognitive skills and practice in contributing to skill development, diminishing returns from cognitive skill, and linear returns from practice.
The skill accumulation equations define how the initial cognitive skill distribution in the individuals translates into an initial bidimensional skill distribution across workers, before their entrance in the job market, and therefore the initial distribution of match values and the rate of skill adjusment while working (on-the-job training). Given this latter feature, the stationary distribution of skills in the model also depends on the parameters of the skill accumulation equation for education, and the planner control variable.

The control variable is the stock of cognitive skill which is devoted to the accumulation of new cognitive skill. Different values of the government control variable define then a technological frontier (or a skill frontier) of possible stationary skills distributions. These parameters - together with the other parameters of the model - are calibrated in order to match the stationary skills and wages distributions with the skills and wages distributions observed in the PIAAC data. Estimating also the planner control within the parameters allows to assess the "actual" design of the educational system and compare it to the one that maximizes aggregate match value at the stationary distribution. In the job search model individuals are endowed with a multidimensional skill bundle that results from the educational system.
The learning process in the educational system can be seen as a transformation of their base cognitive skill level ("at birth") into a two-dimensional skill vector - that measure cognitive and applied marketable skills - dependent on a parameter chosen by the government. Changing this parameter practically corresponds to choosing the weights with which different skills are trained during education.
While entering in the job market they are randomly matched with firms, which have heterogeneous skill requirements along the same dimensions of the individual's bundles. When working, workers adjust their skills adapting to the requirements of the firm matched, producing an instantaneous flow of output (dependent from both the firm's requirements and the worker's skills) subject to a mismatch cost. They can receive outside offers with an exogenous arrival rate from other firms, in a Bertrand environment. The rate of adjustment is endogenous since it depends on the cognitive skills. This is the main difference between other similar job search models Lindenlaub (2014), Postel-Vinay and Lise (2020), Sanders and Taber (2012). By privileging the development of cognitive skills, individuals will be able to adapt more quickly to the requirements of the different firms, at the expenses of lower initial productivity. Therefore, the cognitive skill does not let the individual only to be able to respond to the cognitive skill requirements of the firm, but also allows him to adapt the other types of skills more quickly. This constitutes an additional indirect benefit of cognitive skills, which increases the flexibility of the worker and especially the capability to adapt to labor demand changes and technological innovation. The optimal solution of the model -i.e. the policy that maximizes aggregate value- is characterized by a shift of the initial skill vector towards cognitive skills. Cognitive skills allow to increase the aggregate match value favouring the speed of skill adjustment. Since higher cognitive skills increase the value of job offers with larger initial skill mismatch, the flexibility and the mobility in the job market will increase especially for low-skilled individuals. This increased capability

of adapting to firm's requirements in a perfectly competitive environment decreases the transition costs for a worker (initial skill mismatch), increasing the value of wages, except for workers that have higher baseline cognitive skill, for which the gain in terms of an increased value of external offers is lower than the loss due to initial lower marketable skills. This is due to diminishing marginal returns of cognitive skill in the skill accumulation equation. Further developments of the research include a more detailed analysis of the implications on wages, an analysis on the implications on the capability to adapt to technological shocks in firm's requirements (sectorial shifts), and a more complex form for the production function.

The rest of the paper is organized as follow: in the next section I will illustrate the related literature, from the empirical and theoretical works about learning and the different learning methodologies, both from didactical mathematics and cognitive psychology, to the large literature about educational economics, human capital and job search models; then I will present the theoretical model and the dataset used for identification; at the end of the article I will present and compare simulations of the model both under the educational policy implied by the data and the policy that maximizes aggregate value, discussing the results and the relative implications.

3.2 Literature Review

This review navigates through interconnected domains of economic and educational theories, striving to amalgamate the rich insights from human capital theory, educational economics, labor market dynamics, and cognitive psychology.

Foundational to our understanding of educational investments is the Human Capital Theory, largely heralded by Schultz (1961) and Becker (1964). They articulate the intrinsic value of investing in human capabilities as a catalyst for economic expansion. Further down this line, the models of economic growth presented by Romer (1990) and Lucas (1988), underpin the influential role of knowledge and skill accumulation on economic development, emphasizing the pivotal impact of human capital on proliferating innovation and growth. While the presence of skill accumulation during education resemble Lucas model, it is important to underline that Romer and Lucas cast light on the macroeconomic implications of skill accumulation and investment in human capital, seen as amount of resources allocated in the education (external efficiency), while my exploration investigates the optimal allocation of given resources within the educational system (internal efficiency) and its subsequent micro-level outcomes in the labor market.

The notion of "internal efficiency" in education explores the dialectic between educational inputs and their resulting outputs, primarily in the form of acquired skills and knowledge Coleman (1968). Studies such as Dearden et al. (2002) and Heckman (2007) explore the economic returns to education, whereas our inquiry delves into the nuanced mechanisms through which educational investments transmute into cognitive skills and competencies.

The theoretical part with the job search model, it is inserted directly into the literature that studies the allocation of multidimensional skill bundles and the determinants of wage dispersion through theoretical search models, with the difference that my focus is on the impact of educational policy on different job market outcomes, more than the determinants of wage dispersion. A new emphasis on the roles of both quantity and quality of human capital in the development process, moreover, has given policy makers new appreciation of the importance of education–labor market linkages. The role of the quality has been much less studied than employment outcomes, particularly in developing countries, and is thus less understood. Perhaps degrees attained by young people have greater weight during the school to-work transition (Allen and van der Velden (2007), whereas skills and knowledge prove more important in the long term. However, if the skills acquired in education relate to a very specific occupation, technological change could make these obsolete. The focus on technological development that increases the complexity of jobs has been highlighted in Machin and Reenen (1998). In particular, they highlight the increasing speed in the change of the skill demanded in the job market. Attempts to model the allocation and pricing of heterogeneous supply and demand of indivisible and multi-dimensional bundles dates back at least to Tinbergen (1956) and the hedonic model of Rosen (1974). Recently Lindenlaub (2014) estimates the quadratic-normal assignment model of Tinbergen (1956) along two dimensions of skills (manual and cognitive) for two different cohorts using a combination of O*NET and NSLY data to estimate the distribution of skill requirements conditional on worker's skill bundle. She finds an interesting pattern of technological change: the complementarity between her measures of cognitive worker skills and cognitive job skill requirements increased substantially during the 1990s, while the complementarity between manual job and worker attributes decreased. She then analyzes the consequences of that technological shift for sorting and wage inequality. The same dataset is used also in Lise and Postel-Vinay, a structural model of on-the-job search in which workers differ in skills along several dimensions (cognitive, manual, interpersonal...) and sort themselves into jobs with heterogeneous skill requirements along those same dimensions. Using the above mentioned dataset, the authors used their model to shed light on the origins and costs of mismatch along the cognitive, manual, and interpersonal skill dimensions. The presence of skill accumulation and learning during education is also similar to Sanders (2012), in which the is a two dimensional skill vector of manual and cognitive skills that updates during education with a stochastic component. Other two recent papers are particularly related. Taber and Vejlin (2020) estimate a model which allows for search, human capital accumulation and non wage amenities. Workers are modeled as having a time invariant relative ability at each job-type in the economy. In the absence of frictions they would choose a single job-type and remain indefinitely. Human capital is assumed to be general and accumulated while working. Job mobility is informative about the degree of search frictions, and wage cuts are informative about non wage amenities. In Taber and Vejlin (2016) model relative ability between jobs/occupations as an unobserved vector with dimension equal to the number of job-types in the economy. The aim of my work is to use these setups to study the implications of the choice of the internal composition of the educational programs, presenting a new framework to analyze educational strategies in terms of job market outcomes.

In understanding the translation of educational outcomes to labor market viability, the comparison by Hanushek et al. (2017) between general and vocational education provides valuable insights into the life-cycle labor-market outcomes, further propelling our scrutiny into the relevance and application of skillsets within the labor markets. Similarly, Lamo et al. (2011) illuminates the potential friction and adaptabilities presented by specific skills in the labor market, a concept that is deeply embedded in our exploration of educational efficiencies. A deeper dive into labor market dynamics by Guvenen et al. (2020) elucidates the multifaceted nature of skill mismatch, providing a vital perspective that enriches our examination of educational strategies and their aptitude to navigate such skill discrepancies. Furthermore, Flinn et al. (2017) offers a pivotal foundation for understanding the mechanisms of job search, skill matching, and training in the labor market, accentuating the necessity of correlating educational outputs with labor market demands and aligning skill acquisition with industry requisites.

Enriching this analytical approach towards skill acquisition are the insights from cognitive psychology, particularly by Vaci et al. (2019), which unpack the synergies between intelligence and practice in bolstering skill development. These insights propel our understanding of optimizing cognitive skill development within educational structures and paradigms.

This paper seeks to stitch these varied literatures into a unified analytical framework, intertwining the cognitive development of skills and multi-dimensional labor market dynamics.

3.3 Search Model

The main feature of the framework proposed is to integrate a dynamic of skill accumulation during education to a job search model with multidimensional human capital. The goal is to study the implications that different educational policies (characterized by the stock of cognitive skill that is allocated to the accumulation of different components of the skill vector) have on the mobility in the labor market and on the flexibility of workers in adapting their skills in response to market demand changes and technological development. The environment of the job search model is similar to Postel-Vinay and Lise (2020) and Postel–Vinay and Robin (2002).

3.3.1 Environment

The model is made by a continuous of agents endowed with a two-dimensional skill bundle $x_0 = (x_{0C}, x_{0A})$, which take values in the set of possible skills $X = (X_C \times X_A) \subset R^2$, as in Postel-Vinay and Lise (2020).
x_C represents cognitive skills, while x_A represents applied skills. With cognitive skills, similarly to the other job search models, I refer to what is broadly defined in cognitive psychology literature as "fluid intelligence", that mental capability which depends minimally on prior learning and consists in the ability to formulate abstract mental models to be applied in different circumstances, and that therefore it allows to understand different contexts more easily. By applied skill, on the other hand, I refer to the set of tools, acquired notions that translate into marketable knowledge, and that are directly applied in the labor market and used for production. Knowing how to draw up a balance sheet, how to use a particular econometric technique, or how to make a table all fall within the broad concept of applied skill, even if some of this tasks are "mental activities" and a different level of cognitive skill is necessary to acquire those capabilities. Cognitive skills are then used to accumulate applied skills, that are used in production.

In the future, it is possible to extend the model allowing for more dimensions of applied skills (for example, to separate manual skills) in order to analyze more deeply some phenomena such as job polarisation, but for the purposes of this article I will keep a simpler bi-dimensional framework, since the main focus is on the role of cognitive skills as "technology" in the accumulation of the skills that are used in production, and the trade-off between how much to improve this "technology" by training cognitive skills and the accumulation of applied skills before entrance in the job market. The use of a similar two-dimensional skill vector is present also in Lindenlaub and Postel-Vinay (2016) and Lindenlaub (2017). Individuals accumulate their skills during the educational period at a first glance, and then they enter in the job market. While working, they will adjust their skill to the requirements of the firm matched. The time in which they enter in the job market is an exogenous parameter and will be denoted by t_1. It is exogenous since the focus of the article is not to analyze heterogeneity in educational choices, and moreover the time of compulsory education is fixed. Government may decide to change it, so possible extensions of the model can consider t_1 as an additional policy variable. Once workers are in the job market, they are matched with a firm whose requirements y are drawn from the sample distribution $H(y)$ which take values in the set of possible skill vectors $X \subset R^2$.

When they are matched with a firm, they produce an instantaneous flow of output $p(x,y)$ and adjust their applied skill x_A to the firm requirement as in other search models with on-the-job-training, with the difference that the rate of adjustment will depend on cognitive skills, that will affect the value of the match even if only applied skills are directly used in production. Cognitive skills level will be determined after the first stage of skill accumulation during the educational period. The assumption that cognitive skill is not increased after the educational period is supported by the extensive evidence showing a slow decline-that can at most be stopped- in cognitive skill after the age of 20 de Chastelaine et al. (2011), Morrison and Hof (1997), Persson et al. (2006), Salat et al. (2004). I considered this assumption quite realistic both given this empirical evidence and the context of this model. Given the definition of applied and cognitive skills, firms do not have requirements in term of cognitive skills, so there won't be any skill adjustment for them. It is not common to hear about firms asking for minimum IQ levels as requirements. Maybe they can ask for competencies that are difficult to acquire in terms of cognitive skills effort, and thus more easily acquired for individual with high intelligence, but this is in line with the interpretation of this model of cognitive skills being the "technology" in learning.

The equation for skill accumulation during education depends on initial cognitive skills and practice. As mentioned in the introduction, findings in the cognitive/learning psychology literature (Vaci et al. (2019)) demonstrated a joint effect of intelligence and practice - i.e. that the most intelligent benefit most from the practice - they also estimated the returns from each of the two inputs, also restricting the analysis to the sub-sample considered. The evidence gathered by the authors suggests diminishing returns for intelligence, and constant returns for practice. A similar formulation, with diminishing returns from skill accumulation and linear returns from practice, is present also in Sanders (2012), but with a stochastic component. The skill accumulation equation proposed in this work can be seen as a deterministic version of the latter. In this model, "practice" is interpreted as the share of the stock of cognitive skills allocated at each time in the accumulation of cognitive or applied skills (denoted by $s_i(t)$), and it is the policy variable. Being x_i, $i = A, C$ respectively applied and the cognitive skill, the functional form for the learning equation is as follows:

$$\dot{x}_i(t) = x_C(t)^{\alpha} s_i(t) \tag{3.1}$$

In practice, the skill change in one period of time is a Cobb-Douglas with initial cognitive skill and amount of specific practice for that skill in that period as inputs. The equation above is calibrated with the PIAAC data as explained in the Data and Identification section.

As previously defined, $s_C(t)$ (and $s_A(t) = 1 - s_C(t)$) are the weights according to which cognitive and applied skills are trained during education, so that $s_C(t)$ is the policy variable. So, $s_C : R^+ \to [0,1]$ is the control of the government, and $S = \{s : R^+ \to [0,1] | s \in L^{\infty}[0,1]\}$ the set of possible controls.
Thus, $x^i(x_0^i, y, s_C, t)$ is a two dimensional vector representing the value of the skills at time t given the initial level of cognitive skills x_0^i, the requirement of the matched firm y, and the choice of the government for the policy variable s_C. The initial value of applied skills is zero since they are, by definition, acquired from the external during education and work.
Since the inputs (x_0, y) are random variables, defined in the first paragraph, $x^i(x_0^i, y, s_C, t)$ is the composition of a deterministic function with random variables and is thus a stochastic process. To simplify the notation in this part I will denote the trajectory $x^i(x_0, y, s_C \cdot) : R^+ \to R^2$ obtained for a specific realization of the random variables (x_0, y), and a specific choice of s_C ,simply as $x^i(t)$.

This trajectory, during the period of education, so for $0 \le t \le t_1$ solves:

$$\begin{cases} \dot{x}_C^i(t) = x_C^i(t)^{\alpha} s_C(t), \;\; x_C^i(0) = x_0^i \\ \dot{x}_a^i = x_C^i(t)^{\alpha} s_A(t), \;\; x_A^i(0) = 0 \end{cases} \tag{3.2}$$

The parameter α is calibrated matching model skills ditribution with the sample skills distribution in the PIAAC data.

The law for the period of work is similar to Postel-Vinay and Lise (2020) and other models of on-the-job training, so that workers adjust skill linearly with respect to firm requirements, but the rate of skill adjustment depends on the cognitive skill (in other cited models is an exogenous parameter).

In this period (from t_1) skills are adjusted according to the following ODE (ordinary differential equation):

$$\begin{cases} \dot{x}_C^i(t) = 0 \\ \dot{x}_A^i(t) = x_C^i(t_1)^{\gamma} \max k_m(y - x_A, 0) \end{cases} \tag{3.3}$$

This relation between rate of adjustment, cognitive skills, and mismatch is assessed using PIAAC data, as will be explained more in detail in the next section, together with the rest of the identification strategy, and the above equation is estimated to find coefficients k_m and γ. Assuming a linear adjustment of worker skills in response to firm requirements provides a simplified yet effective representation for several reasons. First, it offers a straightforward and intuitive foundation for modeling, facilitating interpretation and analysis. Second, many job training programs historically adopt a linear progression, indicating that workers often develop skills in a predictable, incremental manner. Lastly, from an economic perspective, firms tend to favor consistent and clear skill trajectories for efficiency, making a linear assumption a reasonable starting point. While this linear framework may not capture every nuance, it serves as a robust initial approximation in understanding worker-firm dynamics and is therefore assumed also in other job search models.

Both equations admits a closed form solution, in particular (3.2) can be solved separating variables in the first equation and then solving it by direct integration,and then substituting in the second equation that can then be solved by direct integration. (3.3), indeed, is constant when $x_A \geq y$ and can be solved directly applying the standard general integration formula for linear ODE's when $x_a \leq y$. The derivation of these solutions will be explained more in detail in the appendix. Consequently, such closed form solutions I can write the specification for the stochastic process x^i as :

$$x^i(x_0, y, s_C, t) = \begin{cases} ((x_0^i)^{1-\alpha} + S_C(t))^{\frac{1}{1-\alpha}} \\ \int_0^t (x_C^i(s))^\alpha s_A(s) ds \end{cases} \quad 0 \leq t \leq t_1 \begin{cases} x_C^i(t_1) \\ \max(x_A^i(t_1), (y - e^{x_C^i(t_1)^\gamma (t-t_1)}(y - x_A^i(t_1))) \end{cases}$$

t$\geq t_1$(3.4)

Where $S_A(t) = \int_0^t s_A(s) ds$ and $S_C(t) = \int_0^t s_C(s) ds$.
Now consider the dynamic of x_A in $[0, t_1]$. First, recalling that $s_A(t) = 1 - s_C(t)$ the above specification can be rewritten (for the interval $[0, t_1]$) as:

$$x_A^i(t) = \int_0^t (x_C^i(s))^\alpha ds - \int_0^t x_C^i(s)^\alpha s_C(s) ds \tag{3.5}$$

Where the last term is equal to $x_C(t) - x_{0C}$ from (3.2), while the term in the middle, again by (3.2) is the value of $x_C(t)$ in the case in which the choice for $s_C(t)$ is $s_C(t) = 1$ for all t in $[0, t_1]$, that is the maximum possible choice for s_C. Always according to (3.2), in this case we would have $S_C(t) = t_1$ and thus

$x_C(t) = ((x^i_{0C})^{1-\alpha} + t_1)^{\frac{1}{1-\alpha}}$. Then, we can rewrite (3.5) as:

$$x^i_A(t) = ((x^i_0)^{1-\alpha} + t_1)^{\frac{1}{1-\alpha}} - x_0 - (x_C(t) - x_0) = ((x^i_0)^{1-\alpha} + t_1)^{\frac{1}{1-\alpha}} - x^i_C(t) \tag{3.6}$$

Given the initial bundle of skills x_0, the skill bundle at the end of the educational period - at time t_1- is deterministic as $x(t_1)$ can be computed as in (3.6) once the planner has chosen its control.

Consider so the "education" map $J : S \times X \to X$, defined as

$$J(s_C, x) = \begin{cases} J_C(s_C, x_C) \\ J_A(s_C, x_A) \end{cases} = \begin{cases} ((x^i_0)^{1-\alpha} + S_C(t_1))^{\frac{1}{1-\alpha}} \\ (x^i_0)^{1-\alpha} + t_1)^{\frac{1}{1-\alpha}} - x^i_C(t_1) \end{cases} \tag{3.7}$$

J maps the initial cognitive skills distribution of the population into a bi-dimensional skills distribution of workers at the end of the educational period.

Since the action of the government is relevant in the model only because of the values determined for the skill bundle at t_1 we can see from (3.7) that the government choice will affect individual skill's evolution only through the term $S_C(t_1)$. We can therefore assume that the government is not choosing a control in a set of functions, because it is enough for it to determine a real number $S_C(t_1)$ in $[0, t_1]$. This will simplify a lot the optimization problem that I will present in the next section, because it will be a one-variable optimization problem instead of a dynamic programming problem. So we can consider the function J as a function of the government's control $S_C(t_1)$ and of the initial skill bundle, that from now on we will denote simply by s.

So J defines a frontier for the initial skill distribution of workers, with any point of the frontier being a different initial distribution of skills depending on the educational design chosen by the government.

We can adopt the substitution $t' = t - t_1$ and consider as starting time the time of entry in the job market. In this way, the skill process will follow (3.4) and the initial condition will be given by $J(s, x_0)$. I will without loss of generality normalize time and assume $t_1 = 1$. So adopting the above mentioned shift in time and using the closed form solution in (3.6) we can write the process for the evolution of individual i skill bundle from the time of entry in the job market (which is now time 0) as:

$$x^i(s,t) = \begin{cases} J^i_C(s) \\ \max(J^i_A(s), (y - e^{(J^i_C(s)(t)^\gamma}(y - J^i_A(s))) \end{cases} \tag{3.8}$$

Where:

$$J^i(s) = \begin{cases} J^i_C(s) \\ J^i_A(s) \end{cases} = \begin{cases} ({x^i_0}^{1-\alpha} + s)^{\frac{1}{1-\alpha}} \\ ({x^i_{0C}}^{1-\alpha} + 1)^{\frac{1}{1-\alpha}} - ({x^i_0}^{1-\alpha} + s)^{\frac{1}{1-\alpha}} \end{cases} \tag{3.9}$$

This defines the set of possible initial skill distributions of workers. The simulation of the model converges to a stationary distribution that is sensible to the initial skills distribution, given the endogeneity of cognitive skills in the process of on-the-job-training that affects match dynamics and wage trajectories. Then the government policy variable defines a technological frontier of possible stationary skill distributions. The actual point of the frontier and the other parameter of the equations in the model will be identified using PIAAC dataset -matching stationary distributions of skills with sampling distributions -and compared with the optimal point that maximizes the aggregate match value in the job market.

3.3.2 Production Function and Value of the match

As explained above, once paired with a firm the worker will produce an instant flow of output that will be directly proportional to both the skills of the individual and the requirements of the firm (so that the firms with the highest requirements are the most productive), but that will be penalized by a mismatch cost. Hence the form for $p(x,y)$ is:

$$p(x,y) = f(x,y) - c(x,y) \tag{3.10}$$

Where

$$f(x,y) = x_A y_A \tag{3.11}$$

while the mismatch cost

$$c(x,y) = k_A \max(y_A - x_A, 0)^2 \tag{3.12}$$

So that the cost mismatch will arise only if the worker is under-qualified. As previously discussed, applied skills include all the competencies that are used in production, and cognitive skills is the technology needed to acquire those competencies, so the output flow will depend (directly) only on applied skills. The simple shape of this dependency is chosen to simplify the tractability and the possibility of analyzing a closed form solution. Hypothesizing a more complex functional form, perhaps with a non-linear dependence on job's skill requirement, could maybe be more realistic but would have little impact on the model given that this does not aim to analyze dynamics linked to the level of production in absolute terms, but more concerning the allocation of different skill bundles and the trade-off between cognitive and applied skills. As long as the production function depends only on applied skills, therefore, the core of these trade-offs is not altered. While working, individuals meet other firms with instantaneous probability λ. The transition rate λ is exogenous and retrieved from the PIAAC dataset. Firms compete à la Bertrand, workers and firms are risk neutral and have discounting rate β. Let P(x,y) be the total value of the match between an individual with skill bundle x matched with a firm with requirements y. If we assume for simplicity that unemployment flow is zero, then the value of being unemployed is just zero and P(x,y) is the surplus value from the match. The worker's value from a match is given by W, where $W \leq P(x,y)$ (otherwise the firm will not be in the match) and $W \geq 0$ (otherwise the worker will be better of quitting into unemployment. This surplus will be shared between the firm and the worker according to the sequential auction model as in Lindenlaub (2017) and Postel-Vinay and Lise (2020). In the sequential auction model, firms offer take-it-or-leave-it wage contracts to workers. When a worker receives an outside offer,the current and outside employers Bertrand-compete for the worker. So, if a worker currently in a match valued P(x,y) with a type-y firm gets an outside offer from a type-y' firm, whose match value would be P(x,y'), we have three cases:

- $P(x,y') \geq P(x,y)$, and the worker accept the offer becoming employed with type-y' firm with wage value W=P(x,y);
- $P(x,y) \geq P(x,y') \geq W$, and the worker stays in the initial match rebargaining the wage to a value W=P(x,y');
- $P(x,y') \leq W$, and the worker stays in the initial match without changing its wage value.

It follows that the match value P(x,y) solves:

$$\beta P(x,y) = p(x,y) - \mu P(x,y) + \nabla_x P(x,y)\dot{x}(x,y) \tag{3.13}$$

That is, the annuity value of the match is equals to the output flow $p(x,y)$, minus the expected loss from job destruction (rate μ), plus the increase in value due to skill adjustment, in which the marginal change of the skill vector is given by (3.3). Notice that the match value is independent by the expected value of future job offers. Future job offers will only affect the sharing of match surplus. This is a direct implication of Bertrand competition. If a worker gets an outside offer with higher value, in fact, it will change job leaving a vacancy worth zero and taking all the value from the previous match, as argued above. If the match value of the outside offer is indeed less than P(x,y), the worker will stay in the match. In any case, the continuation value of the match is still P(x,y). The partial differential equation (PDE) (3.13) admits a closed-form solution which is given by:

$$P(x,y) = \frac{y_A(x_A + x_C^{\gamma}(y_A - x_A))}{\beta + \mu} - \frac{k_A(y_A - x_A)^2}{\beta + \mu + x_C^{\gamma}} \tag{3.14}$$

In the case in which $x_A \leq y_A$ and $x_C \leq y_C$. If one (or both) of this inequalities is not satisfied, and so the worker is overqualified or perfectly qualified, the mismatch cost will just be zero and we'll have $P(x,y) = x_A y_A$, since there will not be on-the-job training. The derivation of this closed-form solution is explained more in detail in the appendix.

Let's look at the match value (3.14): the first term represents the total value from applied skill's output flows, which takes into account a progressive adjustment of a worker's skill toward firm's requirement, while applied skill's mismatch cost (last term) is also discounted for the rate of adjustment (x_C^{γ}), which progressively reduces the mismatch. Higher firm requirements will provide higher match value only if the worker has enough cognitive skills: the first term will increase with the mismatch to an amount proportional to x_C, while the mismatch cost will increase (so the second term of the match value will decrease). Whether the increase in the value of future output flows offsets the increase in initial mismatch will depend on cognitive skills level. This will prevent low skilled individuals to join high requirement firms. Cognitive skills then affect the match value through the progressive reduction of the mismatch. Equation (3.14) clearly shows how cognitive skills will be more important in a given match value when the mismatch is high, making cognitive skills training more valuable for individuals that face on average higher mismatch (low and medium skilled individuals). This latter feature comes from linearity of skill adjustment with respect to job specific requirements. Dropping this assumption - for example assuming that skill adjustment is independent from the current firm/job and the size of the mismatch- will reduce this dynamic for which cognitive skill training is relatively more valuable for medium and low skilled individuals, but not completely, given diminishing marginal returns of cognitive skills in skill accumulation.

3.3.3 Objective function and Match Distribution

Once an individual is matched with a firm, it produces an instantaneous flow of output $p(x,y)$, whose lifetime value $P(x,y)$ is derived in the previous section. As argued before, the private value of the match does not depend on the probability of future job offers, due to Bertrand competition. Once workers are matched with a type-y firm , the match value of worker i is then given by:

$$P^i(J(s,x_0^i),y)$$

Where I recall $J(s,x_0)$ is the skill bundle of the worker with initial cognitive skill x_0 after the educational period conducted with the policy s. Thus, we can compute the expected value generated by the individual i with the initial skill bundle x_0 taking the expectation with respect to firm requirements:

$$EP^i(s,x_0^i) = E_X[P^i(J(s,x_0^i),y)] = \int_X P^i((J(s,x_0^i),y)q(y)dy \tag{3.15}$$

Where q is the density of y given x_0, i.e. the probability for the worker with initial skill vector $J(s,x_0^i)$ of being matched with type-y firm. Since EP^i varies over individuals due to their different initial skill bundle which characterize the individual, we can consider EP^i as a function of the initial skill bundle, defined as $EP(x_0^i,s) = EP^i(s)$ The aggregate value produced by the job market integrating $EP(x,s)$ over the set X, given the density $g(x,s)$ of the stationary skills distribution:

$$P(s) = \int_X [EP(x,s)]g(x) = \int_X P^i(J(s,x_0),y)q(y)g(x,s)dxdy \tag{3.16}$$

So that the dynamic optimization problem that the government has to solve is the following:

$$\max_{s\in[0,1]} P(s) \tag{3.17}$$

In order to solve this optimization problem, densities $g(x,s)$ and $q(y)$ are needed. The initial distribution for cognitive skills is chosen accordingly to IQ distribution in population, which is largely recognized to be a normal distribution. Then applied skill is determined with the skill accumulation equation for education, once educational policy is given. Depending on the educational policy, there will be different initial skills distribution and therefore the model will converge to a different stationary distribution. For any value of s, density $g(x,s)$ is then the stationary skills distribution at which the model converges. The value of s that maximizes aggregate match value at the stationary distribution will be compared with the level of s (and thus the skills distribution and total match value) implied by the data. Distribution of firms' requirements, $q(y)$, is retrieved from the same PIAAC dataset. The optimization problem has been solved numerically and will be discussed at the end of the next section, but from the analytical form of the objective function it is possible to assess some implication. Considering equation (3.16) and substituting for the match value, we obtain:

$$P(s) = \int_X \left(\frac{y_A(x_A + x_C^\gamma(y_A - x_A))}{\beta + \mu} - \frac{k_A(y_A - x_A)^2}{\beta + \mu + x_C^\gamma}\right)q(y)g(x,s)dxdy \tag{3.18}$$

From the map J that defines the skill frontier at the end of education - equation (3.9)- it follows that increasing s will increase initial cognitive skills and decrease initial applied skills. However, the expression for the match value (3.14) shows how the overall impact for a worker will depend on the size of the mismatch he is facing. The higher is the average mismatch, the more the decrease in applied skills will be compensated by the increase in the rate of adjustment. Moreover reduction of mismatches will allow low and medium skilled workers to work for higher requirement firms, and thus, given the dynamic of skill evolution in equation (3.8), their applied skills will converge to higher levels at the stationary distribution. This dynamic, together with diminishing return of cognitive skills in skill accumulation (equation (3.1)) makes an increase in s to change the stationary distribution $g(x,s)$ towards higher levels of both cognitive and applied skills for initially less gifted individuals, and towards slightly higher levels of cognitive skills but lower levels of applied skills for more talented individuals. So it will reduce dispersion in the stationary distributions of skills. An increase in s has the negative effect of reducing skill level and productivity for highly skilled individuals, and initial productivity of all the workers. But it also has the positive effect of allowing low and medium skilled individuals to work for more firms (increasing relative value of outside offers with greater mismatch) and thus being able to increase their skills to higher levels, thanks to higher average cognitive skills (and thus flexibility).

3.4 Data and Identification Strategy

3.4.1 Dataset

The dataset used is the Italian subset of the Survey of Adult Skills, conducted as part of the Programme for the International Assessment of Adult Competencies (PIAAC). This international survey is conducted in over 40 countries/economies and measures the key cognitive and workplace skills needed for individuals to participate in society. The Italian survey that I used consisted of 2209 observations (after cleaning for missing values), with questions on the workplace, job status, job history, actual job's requirements, skill mismatch and competencies in different tasks and cognitive skills test (both for literacy and numeracy). I used this data to estimate the job transition rate, the firms requirements distribution, the equations for skill accumulation both during education and on-the-job and the technological frontier on workers' skill in which the government acts its control. First I summarize how I measured the main variables from the dataset:

- Cognitive Skill. In order to measure x_C, I took the average result of the verbal and numerical reasoning tests performed by the individual during the survey.

- Cognitive Skill Requirements. To measure the cognitive skill requirements of firms, I used questions regarding the quantity and difficulty of cognitive skill tasks performed at work.

- Applied Skill Requirements. In order to assess the applied skill requirements, I used ISCO (International Classification of Occupations skill) level classification. In the ISCO context, skill is defined as the ability to carry out the tasks and duties of a given job, so it fits the definition that I gave of Applied Skills.

- Applied Skill. Once a grid for possible applied skills level is defined using ISCO categories, Applied Skill is determined as follows: exploiting questions about competences mismatch, and given the ISCO classification of an individual's job, the individual Applied Skill is determined as equal to the firm requiremnt if the worker is perfectly matched; one level below if the worker is underqualified; two levels below if the worker is seriously underqualified; and one level above if the worker is overqualified.

- Rate of Skill adjustment. The rate of skill adjustment was measured using a particular Index constructed by PIAAC, "Index of Learning at work" which measure learning new things from supervisors or co-workers; learning-by-doing; and keeping up-to-date with new products or services.

- For the job transition rate I estimated the yearly rate of job change from the history of previous jobs.

Once I obtain measurements for the above variables, matching the stationary distribution for skills in the model with the sampling distributions observed in the data allows to estimate the parameter of the education skill accumulation function and the implied government control value; while studying the correlation in the data between cognitive skills and learning index I modeled the equation for on-the-job training.

3.4.2 Estimation

Considering equations in (3.9), assuming an ex-ante normal distribution for cognitive skills, we have that for any given α and s we will converge to a different stationary distribution for applied and cognitive skills in the job market. Having derived a sampling distribution for these skills from the data presented above, I was able to calibrate numerically α and s in the way that better fits the data.

To calibrate α and s, a grid search approach was employed. This technique involves systematically exploring a predefined range of potential values for both parameters, evaluating stationary distributions implied by each combination of (α, s) against observed distributions of skills. For each combination, the model generates distributions which are compared to empirical data. Then, the combination (α, s) that provides the least sum of squared errors (summing squared errors for both skills dimension) is chosen:

$$(\alpha, s) = argmin \sum_i (x_A^i data - x_A^i(\alpha, s))^2 + (x_C^i data - x_C^i(\alpha, s))^2 \tag{3.19}$$

The range of potential values is (0,1) for both the variables. For s it comes from its definition as explained in the previous section, while for α it is the set of possible values consistent with diminishing returns of intelligence in learning and so with the empirical literature cited in this article. The results of the calibration are summarized in the table below, where the Average Squared Error (ASE) is reported as a measure of goodness of fit.

alpha	s	ASE
0,17	0,22	0,0205

In this way, I calibrated the coefficient α for skill accumulation during education, and I measured the actual policy implied by the data, in order to compare it with the optimal policy that maximizes aggregate value. The skills distribution under the actual policy is reported below:

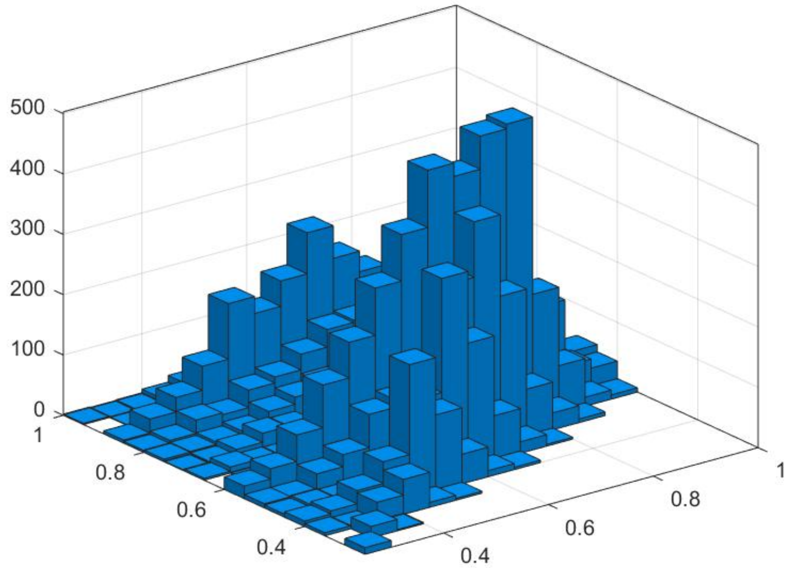

Figure 3.1: Joint skill stationary distribution for s=0.22, $\alpha = 0.17$

3.4.3 Estimation of the rate of skill adjustment

In equation (3.3) I considered the rate of skill adjustment to be proportional to x_C^γ and the size of the mismatch. To validate this functional form and estimate γ and k_m in equation (3.3) I modeled the relationship between rate of learning on the job, cognitive skills and mismatch using the corresponding variables in the dataset. Given z, the rate of skill adjustment measured with the "Index of Learning at work", I conducted the following regression to estimate (3.3):

$$\log(z) = \log(k_m) + \gamma \log(x_C) + \log(y_A - x_A) \tag{3.20}$$

Obtaining the following results:

	log IQ
Coeff	0,36***
S.E.	0.11
p-value	0.002

The results provide a measure for the gamma coefficient to be used in simulating the model and are in line with the aforementioned empirical evidence on the joint effect of intelligence and practice on skill development.

3.4.4 Simulation algorithm and Results

The model solution and simulation has been run on Matlab. The parameters have been chosen accordingly to the previous section, as the coefficient α for the educational map (3.9) and the coefficient γ for the rate of skill adjustment, as previously discussed. Starting from a cohort of 2209 agents, with normally distributed initial cognitive skill, and given the government's choice s, (3.9) is used to compute the initial skill bundle. Then, for every individual, using the closed form solution (3.14), it is possible to compute the match value of the individual for any firm type, and thus to take its expectation with respect to firms requirements. Then, integrating over all the individuals (3.16), $P(s)$ is obtained. Then, $P(s)$ is computed for any possible value of s in a grid from 0 to 1, and the maximizer is chosen. A simulation of the model is not needed in order to find the solution, but I have simulated it up to convergence to a stationary distribution for wages and skills, both with the optimal control and the control estimated in the previous section to match the data ($s = 0.22$), in order to study the dynamics induced by the optimal choice. The optimal choice for the planner given by the algorithm corresponds to $S = 0.3878$, so a significant increase in the practice weight for cognitive skill.

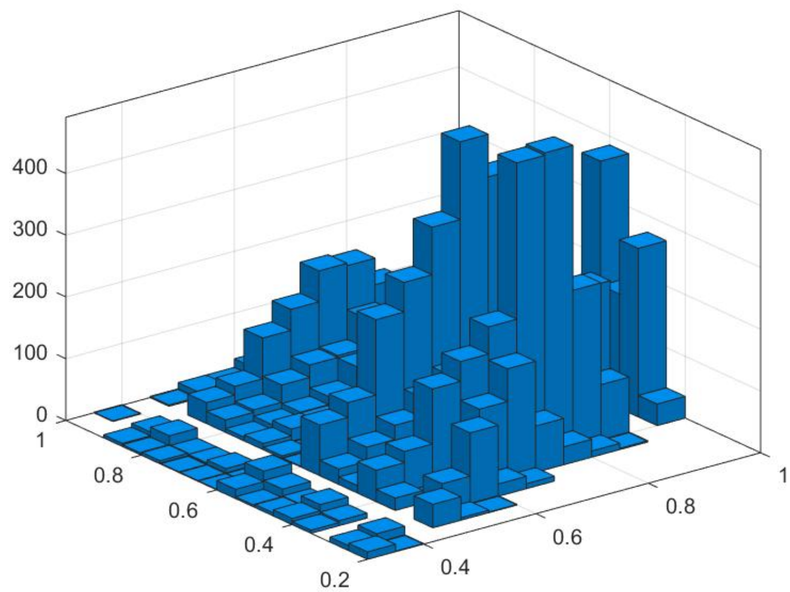

Figure 3.2: Joint skill stationary distribution for the optimal planner choice s=0.3878

As we can see, the overall distribution is shifted towards higher values of cognitive skill. However, the proportion of high cognitive-skill individuals which have medium-low levels of applied skill significantly increases. Due to the initial lower applied skill, indeed, more individuals are prevented from choosing top firms in terms of applied skill requirements, due to an higher initial mismatch cost. This prevents adjustment to top applied skill levels for many gifted individuals. On the other hand, the average applied skill for medium and lower cognitive skills individuals is higher. Of course, increasing initial cognitive skills facilitates the acquisition of future skills, to the detriment of the initial applied skills. The individual will therefore have less specific knowledge (and will be less productive initially) but at the same time will be faster in learning. If the burden in learning cognitive skills during education is too high, the lack of initial productivity will be too high to be compensated for by its future increase. Given the diminishing returns of cognitive skill in learning, this increase in general cognitive skill has major benefit for less skilled workers. Their mobility is increased and so is the ability to switch to more productive firms. For the most skilled workers the loss in initial productivity is not compensated by the lower gain they have in terms of increased cognitive skill (diminishing returns), and thus the match value with top firms will be more affected by the decrease in their initial applied skill than by the slight increase of their cognitive skill. For this reason, as pointed above, they may be in a position to not accept matches with the most productive firms. Moreover, since the probability of encountering them is exogenous, a high cognitive skill does not increase the chances of finishing in a good match. There are therefore two effects of the increase in cognitive skills. On the one hand, initial productivity is sacrificed for higher productivity in the future, and this applies to all types of workers. On the other hand, job mobility and the quality of future matches (and therefore their future applied skill) only increase for workers with medium-low skills. It should be specified that an increase in cognitive skill allows even initially less gifted workers to be able to work in firms with medium-high requirements. In fact, if a worker with a relatively low-medium skill encountered a productive firm, with low cognitive skill policies the mismatch value would be too high, and the individual would not be able to work for that firm. With a cognitive skill development policy, on the other hand, a higher percentage of workers can potentially work for more productive (or important) firms. The initial specific skills, in fact, are in both cases irrelevant for the value of the match which becomes constituted above all by the expectation of future productivity once the worker has acquired the necessary skills and by the mismatch value. Therefore, marginal differences in initial applied skills, for less skilled individuals, are less relevant compared to an increase of the learning rate under the optimal policy. Referring to the match value (3.14), in particular to numerator in the first term capturing the output flow:

$$y_A(x_A + x_C^\gamma(y_A - x_A))$$

when the worker is strongly under skilled, the first addend is not very relevant compared to the second, therefore an increase in cognitive skill is more valuable and can change the value of the match in a decisive way.

Given the diminishing returns of cognitive skill in learning, an increase in general cognitive skill benefits less skilled workers more. As argued, their mobility is increased and so is the ability to switch to more productive firms. An important result, which will be deepened in the future developments of this article, is the general increase in resilience to shocks in the requirements of the firms, and therefore a greater adaptability to technological change. I report the paths for average productivity resulted from the model simulations, conducted both with the planner's optimal policy and the actual educational policy.

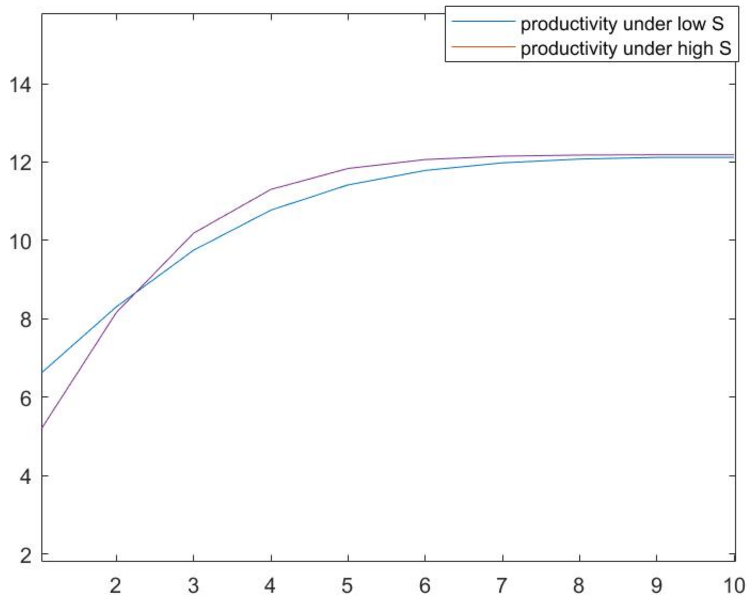

Figure 3.3: Average productivity under the optimal control is initially lower, but it grows faster thanks to the higher rate of skill adjustment

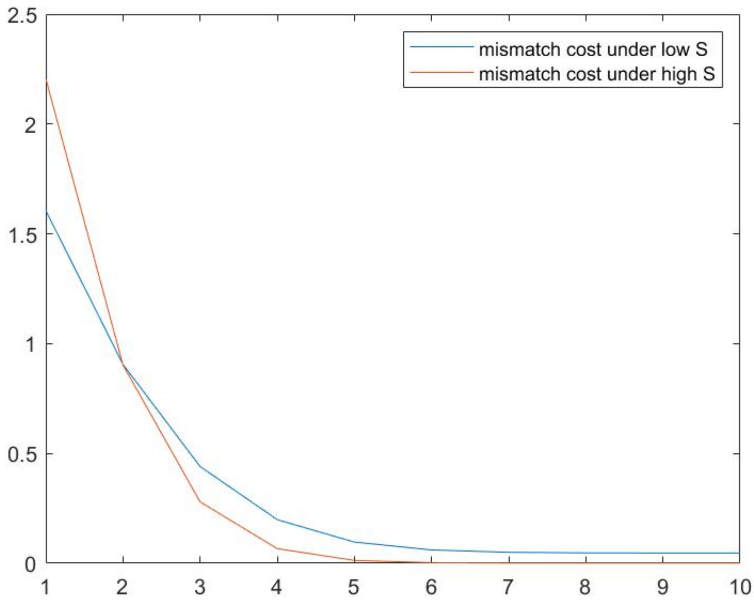

Figure 3.4: Average mismatch cost falls faster thanks to the higher rate of skill adjustment

Establishing a higher practice weight for cognitive skills means reducing the amount of training in applied skills, therefore sacrificing initial productivity in order to obtain a higher skill adjustment rate that allows the individual to be more flexible in learning different skills . This not only increases future productivity and reduces future mismatch cost, but also increases the average mobility and flexibility of the labor market by increasing the ability of individuals to adapt more easily to different skill requirements. In fact, the simulation shows an increase in job mobility and the times in which the salary is renegotiated in case the optimal control is chosen. This is due to the fact that, when the cognitive skill is less trained, less skilled individuals can refuse offers from firms with high requirements because the initial mismatch cost is too high and the time needed to fill it is too much. When the cognitive skill is higher also for this group of individuals, the time required to fill the skill mismatch decreases and therefore offered by firms with high skill requirements are more advantageous even for workers with medium-low cognitive skills. I expect this dynamic to occur to a greater extent if more dimensions are introduced for applied skills. In this case, even individuals with higher cognitive skills could reject potentially more productive companies for the initial skill mismatch, and therefore an increase in the rate of skill adjustment should increase mobility for these individuals as well. Due to Bertrand competition between firms, higher average value from outside offers means higher wage value (the share of value, W ,given to workers). Since the number of times wages are renegotiated, or jobs are changed, increase with optimal control, average wages grow faster.

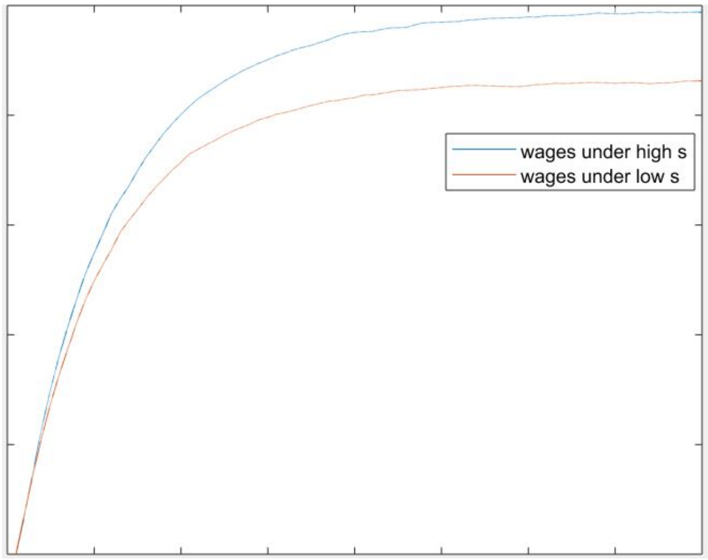

Figure 3.5: Average wage grows faster due to increased job mobility

Before we argued that a higher emphasis on cognitive skills in educational curricula would yield more substantial benefits for individuals who are categorized as low to medium-skilled, as opposed to those who are considered high-skilled. For low and medium-skilled individuals, an increased focus on cognitive skills amplifies their ability to swiftly adapt to new job roles and requirements. This, in turn, boosts their 'applied skill level,' a metric we define as the effective level of skill an individual can apply in their current role. The reason being that these individuals are more likely to find jobs at firms with an overall higher average skill requirement due to their enhanced adaptability. Therefore, they find themselves in environments that allow for greater growth, and consequently, their average skill level rises.

However, for high-skilled individuals, the landscape is a bit more nuanced. While these individuals are indeed smart and adaptable, they face diminishing marginal returns on intelligence in the context of learning and skill adaptation. This results in a curious dynamic; while high-skilled individuals can adapt their skills quickly, they stand to lose more due to a higher average 'initial skill mismatch' when starting new roles. Essentially, the gains from higher adaptability do not necessarily offset the losses incurred from this initial mismatch.

The upshot of this shift towards cognitive skills is evident wage dynamic, particularly under a policy optimized by a theoretical planner aiming for maximum societal benefit. Under such an optimized policy, average wages grow faster due to increased job mobility for low and medium-skilled individuals, facilitated by their enhanced adaptability. In effect, these individuals can more readily switch between roles and firms, seeking out opportunities for higher wages and thereby contributing to a wage distribution that is more tightly clustered around a higher average. It is important to remark that interpreting cognitive skills as technology in learning, and therefore not including them directly in production and in firms requirements and therefore not allowing for on-the-job training for cognitive skills is not only consistent with the empirical findings, as previously discussed, but does not impact the core result: allowing cognitive skills to participate directly in production, in fact, will make these skills even more valuable enhancing the shift of the optimal policy towards higher training of cognitive skills during education.

3.5 Conclusion

This study aimed to present a framework for analyzing the job market implications of an educational system that places greater relative importance on the training of cognitive skills. The educational system is modeled as a transformation of initial cognitive skills in the population into a two dimensional skill bundle composed by cognitive skills after education and applied skills acquired during education and applicable in production (marketable knowledge). Using an equation for skill accumulation in the school system - calibrated using PIAAC dataset- I identified a frontier of the possible skill distributions of workers. Each point in the frontier is a different two-dimensional skill distribution that corresponds to a different design of the educational system chosen by the theoretical planner, that chooses the stock of cognitive skills allocated to the accumulation of applied or new cognitive skills. The environment is similar to common frameworks such as Postel-Vinay and Lise (2020) and Postel–Vinay and Robin (2002) in which individuals are randomly matched with a firm and produce an instantaneous flow of output which is proportional to the workers' skill and the firm technology but has a penalty term due to skill mismatch. Workers gradually reduce the skill mismatch by adapting their skills to firms requirements.
The main difference with respect to similar models is that the rate at which they adjust their skills depends on the cognitive skill, so it is endogenous and depends on the design of the educational system. The theoretical job search model is therefore used in this first basic framework to define internal efficiency within the educational system, to formalize which dynamics can change in the latter by implementing different educational models. In particular, with the model I intend to formally explain how sacrificing learning of applied concepts or tools in exchange for greater mental elasticity can allow to extract greater value from the labor market through increased flexibility and mobility, together with a more egalitarian range of opportunities and a reduction of job polarisation, due to diminishing returns of cognitive skills in skill accumulation and the capability of cognitive skills of reducing the impact of initially larger mismatch.

The results showed that increasing the weight of cognitive skills in education results in lower initial productivity but higher rates of adjustment, particularly for low and medium skilled individuals, leading to increased overall mobility. The optimal solution was found to have a cognitive skills weight in education higher than the current level. Improving overall cognitive skills, especially for low and medium skilled individuals (diminishing marginal returns) allows workers to adjust faster to higher mismatches, and therefore allows also medium skilled workers to arrive at top jobs.

Overall, the findings of this study suggest that increasing the attention to

the training of cognitive abilities in the primary school system can help create a more mobile, flexible and democratic labor market. This is particularly relevant in the context of disruptive technological innovations, which will make cognitive skills much more valuable, particularly for those who were less trained in this area during their education.
The model presents several limitations: the basic form of the production function is suitable for a close form solution which allows to make comparative statics with the explicit match value, but is not calibrated with actual data and its functional form might not fully capture real-world complexities. Moreover, the dynamic of skill acquisition during education is modeled with a general Cobb-Douglas form whose parameter are calibrated matching the workers skill distribution in the PIAAC data with the skill frontier implied by the accumulation equations. Even if in this case the modeling is informed by real data, a general form for the skill accumulation functions does not include all education dynamics. Future iterations of this model will aim to include a more nuanced representation of 'applied skills' and will consider a skills-based firm distribution to provide a comprehensive analysis of labor market mobility and skill mismatch costs. Another important implication of this framework concerns adaptability of workers to technological innovations. An increase in the rate of skill adjustment due to a greater cognitive skill not only allows workers to adapt faster to the skills required by firms, but also to technological changes. Another important development of this work is therefore to analyze the response of the labor market to various shocks in the composition of the requirements of the firm due to disruptive technological innovations. In this environment, major technological changes will make cognitive skills even more valuable, enhancing the most gifted individuals especially if the cognitive skill is not trained in a relevant way during education. The mechanism presented in the model -in which higher cognitive skill training is able to reduce skill polarization- is expected to amplify in this context.

The conclusions drawn from this study carry significant policy implications, especially when considering the evolving landscape of the labor market due to technology-driven changes. The advent of disruptive technologies such as artificial intelligence, machine learning, and automation has already begun to reshape skill requirements across various sectors. In such a fluid context, the ability to rapidly adapt and acquire new skills becomes increasingly critical. Therefore, an educational foundation that empowers individuals with strong cognitive skills is more than just beneficial; it becomes a vital prerequisite for economic resilience and social mobility in the 21st century.

By cultivating these cognitive abilities from an early age, the education system can act as a proactive force, not just preparing students for existing jobs but for roles that have yet to be created. This forward-thinking approach ensures that future generations are not only capable of adapting to new job environments but are also better prepared to become innovators and leaders in fields that may not yet exist. This results in a workforce that is not only more flexible and adaptable but also more creative and forward-thinking, attributes that are essential for driving innovation and staying competitive in a global market. The elevated significance of cognitive skills, thus, has a cascading effect that extends far beyond individual benefits. It reaches into societal structures, potentially reducing inequalities by providing equitable access to opportunities, and even has implications for national economies, making them more agile and better prepared to adapt to global challenges. As we look towards the future, extending this model can provide more granular insights that could guide educational policy, preparing us for a world that is not just rapidly changing, but also increasingly unpredictable.

3.6 Acknowledgements

I would like to thank my supervisor Daniela Di Cagno, for the pleasant collaboration and her guidance, and my family for always supporting me in everything.

3.7 Data Availability Statement

The data that support the findings of this study are openly available in "figshare" at
https://doi.org/10.6084/m9.figshare.22086353.v1.

3.8 Appendix

In this section I will derive the solution for the closed form solution (18) of the PDE

$$\beta P(x,y) = p(x,y) - \sigma P(x,y) + \nabla_x P(x,y)\dot{X}(x,y) \tag{3.21}$$

We will consider the relevant case in which the worker is underqualified with respect to both skills. computing the scalar product in (23) and rearranging we get:

$$(\beta + \mu)P = p(x,y) + \frac{dP}{dx_A} x_C^{\alpha}(y_A - x_A) \tag{3.22}$$

By similarity, we will search for a solution in the following form:

$$P(x,y) = f(x_A, y_A) - \frac{k_A(y_A - x_A)^2}{\beta + \mu + x_C^{\alpha}} \tag{3.23}$$

plugging into the PDE (24) we obtain:

$$f(x_A, y_A)(\beta+\mu) - \frac{(\beta + \mu)k_A(y_A - x_A)^2}{\beta + \mu + x_c^{\gamma}} = x_A y_A - k_A(y_A - x_A)^2 + f'(x_A, y_A)(x_c^{\gamma})(y_A - x_A) - 2\frac{(\beta + \mu)k_A x_c^{\gamma}(y}{\beta + \mu +} \tag{3.24}$$

Computing the above, all left terms simplifies and we are left with;

$$f(x_A, y_A) = f'(x_A, y_A)(x_c^{\gamma})(y_A - x_A) + x_A y_A \tag{3.25}$$

so that the PDE is reduced to a standard ODE. Using Duhamel's formula we obtain the solution for f:

$$f(x_A, y_A) = y_A(x_A + x_c^{\gamma}(y_A - x_A)) \tag{3.26}$$

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Chapter 4

Intuitive Cognitive Abilities and Ambiguity Attitudes: An Experimental Investigation

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Abstract

This study explores the relationship between intuitive cognitive abilities, associated with System 1 in the dual-process theory of decision making, and attitudes towards risk and ambiguity through a laboratory experiment. Central to our approach is the WarmApp activity, a novel computer-based tool designed to assess various domains of intuitive cognition. We employ a two-parameter model to estimate risk and ambiguity attitudes based on responses to experimental tasks. Additionally, our study investigates the potential of WarmApp training as a means to mitigate ambiguity aversion. Results indicate that individuals with higher intuitive cognitive abilities are more tolerant of risk and ambiguity, and that targeted training using WarmApp can effectively reduce ambiguity aversion in individuals with lower innate System 1 proficiency. These insights contribute to the understanding of how intuitive cognition influences economic decision-making and underscore the potential of cognitive training in modifying attitudes towards ambiguity.

Keywords Education, Ambiguity, Risk, Dual Process Theory.

4.1 Introduction

Ambiguity aversion, a concept first introduced by Ellsberg (1961), has become a key concept in understanding decision-making in economics and psychology. It describes the preference individuals exhibit for known risks over uncertainties with unknown probabilities. This behavior is prevalent across various domains, including market participation, election decisions, and career choices, often occurring in situations where probability distributions are partially or completely unknown.

The roots of ambiguity aversion remain a subject of intense study and debate. A growing body of research is exploring the role of cognitive processes, particularly focusing on how intuitive thinking influences one's tolerance for ambiguity. For example, studies have revealed that individuals with a stronger reliance on intuition, as quantified by a survey, are generally less averse to ambiguity Butler et al. (2013).

In line with these developments, our study leverages the dual-process theory of decision-making, articulated firstly by Stanovich and West (2000) and then by Kahneman (2011). This framework delineates two distinct cognitive systems: System 1 and System 2. System 1 operates rapidly and automatically, guiding immediate judgments and instinctive reactions. This intuitive process is associated with intuition and characterized by its speed and lack of deliberate control, operating seamlessly without explicit intention or conscious awareness. On the other hand, System 2 is deliberate and conscious. It engages in a controlled mode of thinking, characterized by methodical, analytical and effortful processing. This system is responsible for thorough, systematic comparisons and requires active cognitive engagement and focused attention.

Klein (2003) firstly associates System 1 with intuition, linking it to behaviors inclined towards adventurousness and risk-taking. This association is grounded in the observation that intuitive judgment often correlates with a propensity for risk-taking and a reliance on improvisation. Research on dual-process theory has relatively underexplored its implications for risk and ambiguity, with notable exceptions such as the evidence presented in the study by Butler et al. (2013).

Our research aims investigates the roles of System 1 and System 2 in the context of risk and ambiguity. By examining how intuitive (System 1) and deliberative (System 2) cognitive processes influence decision-making under these conditions, we seek to provide deeper insights into the cognitive mechanisms underlying economic behaviors in uncertain environments.

The main focus is on the influence of System 1 processes on ambiguity attitudes, and whether System 1 training can potentially mitigate ambiguity aversion, thereby contributing to the broader understanding of the cognitive underpinnings of ambiguity attitudes.

This study extends into another significant domain of our broader research: the relationship between System 1 abilities and mathematical performance. This linkage provides valuable insights into educational strategies, particularly in the realm of mathematics education.

In our other ongoing research, we explore the idea that mathematical performance is closely tied to the inquiry-based process of discovery through trial and error. This concept, articulated by Hintikka, suggests that the journey of mathematical discovery is often paved with risks and uncertainties associated with exploratory approaches Hintikka and Remes (1974). A key aspect of this process is the willingness to engage in trial and error, a method fundamental to inquiry and discovery in mathematics. However, the risk associated with 'taking the plunge' into such exploratory activities can often be a barrier, as the fear of making mistakes might hinder the inquiry process and impede mathematical discovery. This notion is supported by Sharma (2015), who emphasizes the importance of fostering a safe learning environment that encourages risk-taking in mathematics classrooms. Similarly, Lake (2019) discusses the intricate balance between 'playing it safe' and 'throwing caution to the wind,' highlighting how risk-taking and emotional responses play crucial roles in mathematical learning environments. Furthermore, Oyarzun (2013) provides a theoretical foundation for understanding the intersection of learning and risk aversion, which is particularly relevant to our discourse.

The current research, therefore, not only advances the understanding of decision-making in economic contexts but also intersects with our broader research interests in mathematical education. It highlights the pivotal role of cognitive processes in educational settings and underscores the need for pedagogical approaches that integrate an understanding of intuitive and deliberative thinking in facilitating effective learning experiences.

Our experimental approach encompasses several phases. Initially, we engage participants in tasks that measure their aversion to risk and ambiguity. Risk aversion is assessed using the multiple price list format developed by Holt and Laury (2002), while ambiguity aversion is measured through choices involving urns with known and unknown compositions, following the methodology pioneered by Ellsberg. Subsequently, participants take part in the "Warmapp activity," a tool designed by one of the authors (Bernardi L.) OILER (2021) to activate and measure System 1 processing capabilities. This activity includes tasks that test various facets of intuitive cognition, such as the Approximate Number System (ANS) acuity - Dehaene (2011)-, pattern recognition, and object recognition . The design of Warmapp ensures that participants rely on System 1 processing by limiting the display time of each task, thereby preventing the use of more deliberate, System 2 processes like counting or shifting spatial attention. Prior studies, such as Halberda et al. (2008), have already identified a link between higher ANS acuity and improved performance in tasks involving numeric estimation and basic arithmetic .

After the Warmapp intervention, participants revisit the ambiguity aversion tasks, allowing us to evaluate the effect of the training on their ambiguity attitudes. A control group undergoing a placebo task instead of the Warmapp activity is also included to bolster the robustness of our findings. This study is situated within a theoretical model that facilitates the estimation of risk and ambiguity attitudes. The model incorporates choices made in the risk and ambiguity tasks and employs a two-parameter approach to estimate the parameters of the Constant Relative Risk Aversion (CRRA) utility function alongside an ambiguity aversion parameter, as in Halevy (2007) and Gneezy and List (2015).

In our main regression analysis, we explore the correlation between performance in the Warmapp activity and attitudes towards risk and ambiguity. Our goal is to discern whether an enhanced System 1 capacity correlates with reduced ambiguity aversion. Additionally, by comparing the Warmapp and placebo groups, we aim to determine if specific improvements in System 1 processing lead to a measurable decrease in ambiguity aversion.

The implications of our findings are relevant especially for educational policies. Should our hypothesis that Warmapp training effectively reduces ambiguity aversion be confirmed, it could significantly influence future policy-making strategies. Such interventions could enhance decision-making under uncertainty, impacting diverse areas such as economics, politics, finance, and even educational methodologies.

Moreover, this research contributes to the ongoing discourse in educational strategies, particularly in mathematics and science. As Sharma (2015) notes, the propensity for risk-taking and embracing trial-and-error methods is crucial in developing mathematical competence. Our study's exploration into risk and ambiguity attitudes has significant pedagogical implications, especially considering the evolving landscape of student-centric education, where open-ended problem-solving often involves navigating through ambiguity and risk.

The remainder of this article will provide a comprehensive literature review on ambiguity aversion, the dual-process theory, and the specifics of the Warmapp activity. We will detail our experimental design, the methodologies for measuring risk and ambiguity aversion, and the intricacies of the Warmapp activity. The results section will present our findings on the relationship between System 1 processing and ambiguity aversion, along with the impact of Warmapp training. Finally, we will discuss the broader implications of our results, acknowledge potential study limitations, and suggest directions for future research. Our objective is to enrich the understanding of how intuitive cognitive abilities influence ambiguity aversion and to offer insights that could inform effective policy interventions to improve decision-making in uncertain environments.

4.2 Literature Review

Ambiguity aversion, a pivotal concept in decision-making research, was first articulated by Ellsberg (1961). This research domain encompasses a variety of methodologies for eliciting ambiguity and an array of theories to explain this intricate phenomenon. Our literature review aims to highlight the most salient studies in this arena.

Early research emphasized a general preference among individuals for known risks over unknown ones. This behavior significantly influences various domains, including market engagement, selective participation in elections, and career decision-making.

Over time, the methods for assessing ambiguity have evolved. Notably, the works of Gneezy and List (2015) and Pace et al. (2014) have substantially refined our comprehension of ambiguity aversion. Gneezy and List's two-parameter model distinctively delineates the impact of risk and ambiguity attitudes, offering an advanced insight into decision-making under uncertainty.

Additionally, Pace, Hey, and Maffioletti expanded traditional binary-choice tasks with their three-color Ellsberg urn experiment, facilitating a more nuanced understanding of ambiguity attitudes, extending beyond simple aversion to include neutrality and even attraction.

Understanding risk aversion is also crucial, as insights into choices under risk conditions are foundational for interpreting behavior in ambiguous situations. Such insights are integral to our study, especially for analyzing the relationship between risk and ambiguity aversion.

Moreover, there has been burgeoning interest in the cognitive determinants of ambiguity aversion. The research by Butler et al. (2013), for instance, underscores the role of intuitive thinking in reducing ambiguity aversion, suggesting an inverse relationship between intuitive reliance and ambiguity aversion.

Our research further probes into the connection between System 1 cognitive abilities and ambiguity aversion, situated within the dual-process theory framework as defined in Kahneman (2011) and Stanovich and West (2000). Utilizing the WarmApp activity as a measure of System 1 processing, we delve into the cognitive mechanics at play in ambiguity aversion.

The significance of numerical cognition in decision-making under uncertainty, highlighted in studies by Dehaene (2011) and Halberda et al. (2008), underscores the role of the Approximate Number System (ANS) in intuitive processing, forming a crucial backdrop for our use of the WarmApp activity.

The interplay between personality traits, especially the Big Five, and economic decision-making has also been a focal point of research, as outlined in Matthews (2003) and Costa and McCrae (1992). Understanding how these traits influence economic behaviors is important, and our study incorporates personality assessments to provide a more comprehensive analysis of economic decision-making.

Importantly, our discussion extends to the realm of mathematical education and the implications of risk-taking in learning environments. The works of Sharma (2015), Lake (2019), and Oyarzun (2013) highlight the significance of fostering environments conducive to risk-taking and inquiry in mathematics education. This aligns with our broader research interests, where we explore how System 1 processing and its influence on risk and ambiguity attitudes impact mathematical discovery and learning.

In summary, our study not only contributes to existing literature by integrating experimental methods to elicit risk and ambiguity attitudes but also investigates the role of System 1 cognitive mechanisms in ambiguity aversion. By examining the potential of interventions like the WarmApp activity in reducing ambiguity aversion, our research offers novel insights with implications for future policy strategies and educational methodologies.

4.3 Experimental Design

This study employs a laboratory experiment to investigate the relationship between intuitive decision-making (System 1 proficiency) and economic behaviors related to risk and ambiguity aversion. The experiment is designed to obtain measures for participants ambiguity and risk aversion, assess their System 1 Proficiency, train this cognitive process and then re-evaluate their ambiguity attitudes. Moreover, measures of individuals IQ, personality traits and demographics are needed, in order to control for variables that can potentially affect ambiguity attitudes or correlate with the performance in the Warmapp activity.

In this section we outline each step of the experiment detailing how we obtained measures of all variables. First, we conducted tasks with monetary incentives to measure individuals risk and ambiguity aversion, selecting randomly for each participant which task to visualize first in order to avoid order effect. Then participants engaged in the Warmapp activity, repeating it for 15 rounds. The average score of the first three rounds is used as a measure of the individual System 1 Proficiency. At the end of every round participants received feedbacks and nudges to stimulate System 1 thinking, and the rate of improvement in the score between each round is measured. Participants repeat the ambiguity task and fill a questionnaire containing questions on demographics, a UNIT IQ test and a Big Five personality test. At the experiment's conclusion, lottery draws are carried out to determine the payoffs for the tasks assessing risk and ambiguity.

4.3.1 Ambiguity Task

To assess ambiguity aversion among participants, we employed a task inspired by the Ellsberg Paradox. This task involves a series of choices between urns with known and unknown probabilities, and choices will be converted into an ambiguity parameter according to the theoretical framework proposed in the next section. Participants were presented with a sequence of choices involving two urns, A and B. Urn A contained a known distribution of black and white balls, while Urn B contained 100 balls of unknown color distribution. Participants were informed that they would win a fixed monetary reward (5 euros) if a white ball was drawn from the selected urn. This setup was repeated across several lots, with the proportion of white balls in Urn A systematically decreased to observe the point at which participants switch their preference from Urn A to Urn B.

Lot Examples

Lot 1

- Urn A: 50 black balls, 50 white balls.
- Urn B: 100 balls, color composition unknown.
- Choice: Select Urn A, Urn B, or random selection.

Lot 2

- Urn A: 55 black balls, 45 white balls.
- Urn B: As above.
- Choice: As above.

The rationale behind this task is to measure the extent to which individuals are willing to tolerate ambiguity as opposed to quantifiable risk. A preference for Urn A, with its known probabilities, indicates ambiguity aversion, whereas a switch to Urn B suggests a higher tolerance for ambiguity. By varying the known risk in Urn A and observing the switching point, we can infer the participants' ambiguity aversion levels. To ensure the robustness of our findings and avoid any color bias, the color of the winning ball (white or black) was randomized across participants. This randomization was intended to control for any potential preference or perceptual biases associated with specific colors. Participants were informed that at the end of the experiment, one of the lots would be randomly selected and played for the monetary reward. The instructions and choice presentation were standardized to ensure clarity and consistency in participants' understanding of the task.

4.3.2 Risk Task

To measure risk aversion, participants were engaged in the Holt and Laury (2002) task, a widely recognized method for eliciting risk preferences. This task involves a series of binary choices between two lotteries, each with different levels of risk. Each participant was presented with a sequence of 10 binary lotteries. The two lotteries in each pair offered the same probabilities for high and low payoffs, but the variance in payoffs differed. The left lottery presented a lower-risk option (e.g., 5 euros for the high payoff and 3 euros for the low payoff), while the right lottery presented a higher-risk option (e.g., 8.30 euros for the high payoff and 0.30 euros for the low payoff). Across the 10 choices, the probability of the higher payoff increased by 10% each round, from 0 to 1.

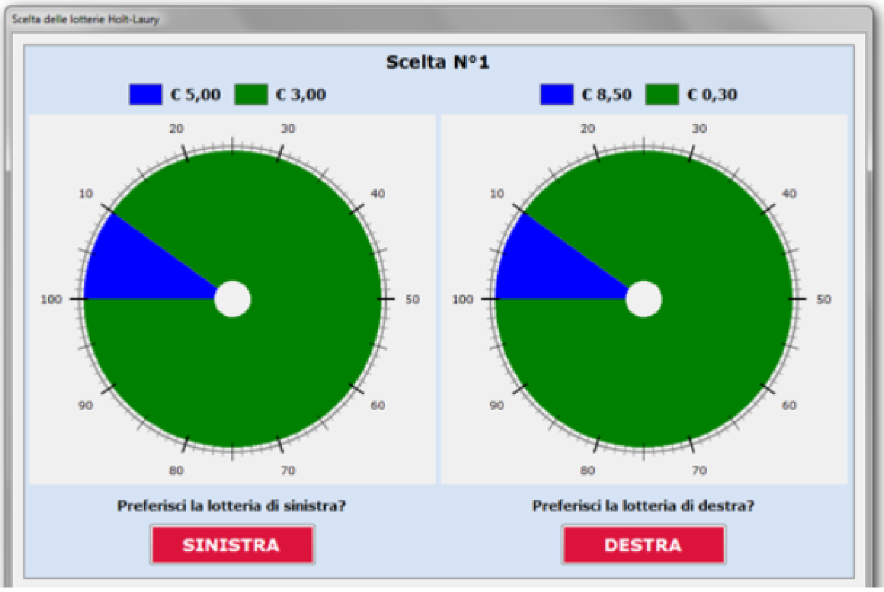


Figure 4.1: First Pair of Lotteries in Holt and Laury Task

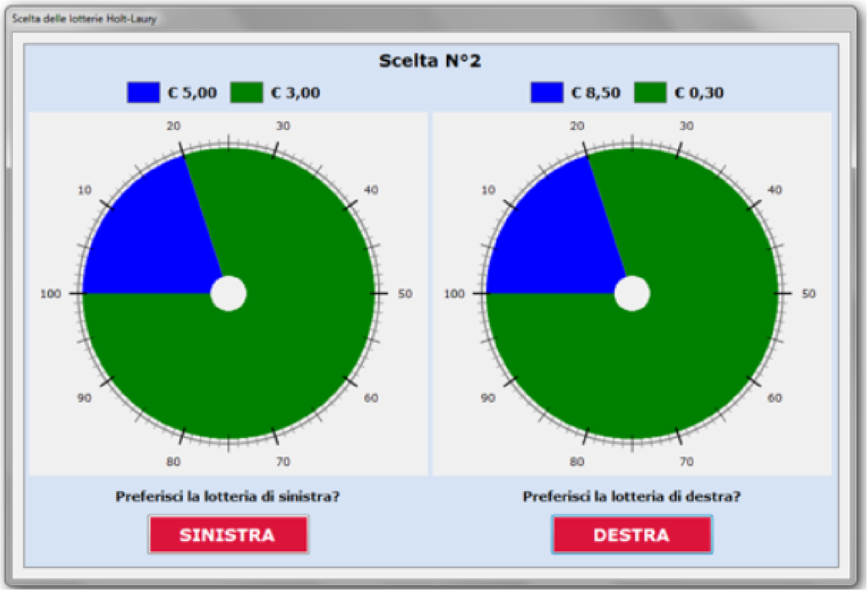


Figure 4.2: Second Pair of Lotteries in Holt and Laury Task

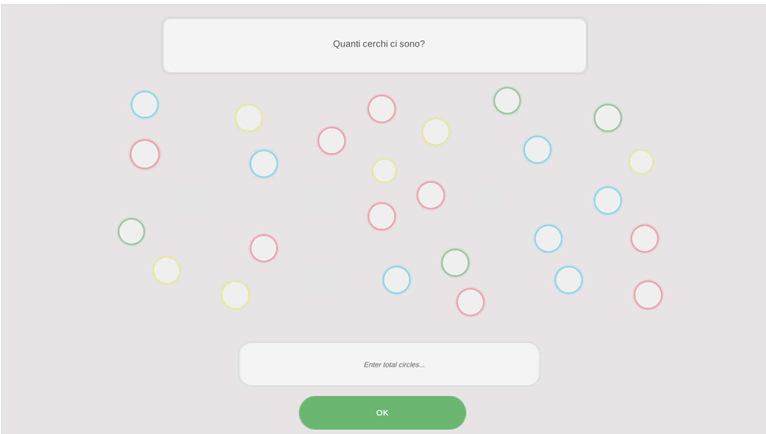
The key measure in this task is the point at which the participant switches their preference from the safer to the riskier lottery. This switching point is used to determine the individual's level of risk aversion. As the probability of the high payoff increases, risk-averse individuals are expected to switch at earlier stages, while risk-tolerant individuals may prefer the higher-risk lottery for longer. Based on the sequence of choices made by each participant, a risk aversion parameter will be determined, adhering to the theoretical framework outlined in next section. This parameter effectively quantifies the degree of risk aversion exhibited by each participant. At the conclusion of the experiment, one of the ten pairs of lotteries will be randomly selected. The lottery chosen by the participant during the task from this pair will then be played out, and the corresponding payoff will be awarded.

4.3.3 Warmapp Activity

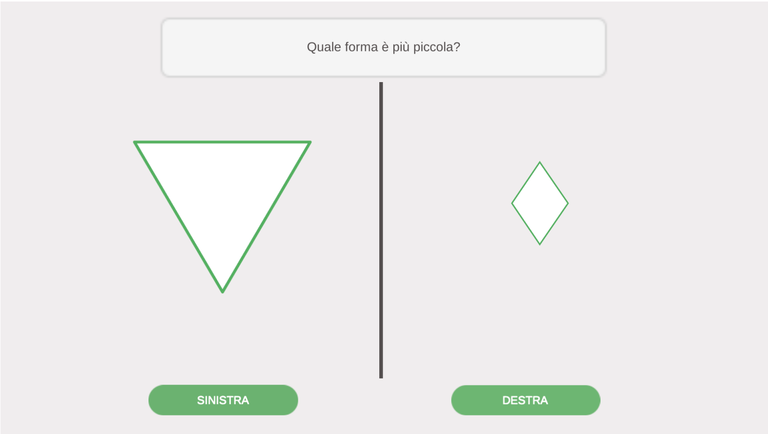
The WarmApp activity, a component of the Oiler educational project, is designed to exercise intuitive judgments in numerical estimation, pattern recognition, and object identification. This computer-based activity comprises a series of mini-games that challenge participants to make quick, intuitive decisions, aligning with the characteristics of System 1 processing as defined by Kahneman's dual-process theory. Each mini-game within WarmApp is structured to ensure decisions are made rapidly, since images are visible just for 1 second, thus minimizing the involvement of deliberate cognitive mechanisms, such as counting, and preventing shifts in spatial attention. This setup aligns with the automatic and fast decision-making processes characteristic of System 1, as these tasks require immediate and intuitive responses. WarmApp consists of five distinct game types, each targeting a different aspect of intuitive cognition:

1. **Numerical Comparison:** Participants choose the higher or lower number of items between two sets. The levels vary in the number of items and the maximum difference between sets.

2. **Numerical Estimation:** Participants estimate the number of items on the screen, with different levels presenting varying item counts. This task also serves to measure ambiguity aversion.

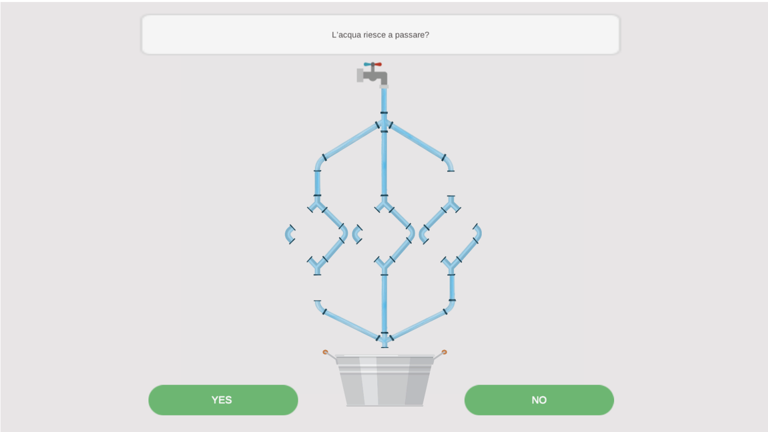


3. **Shape Comparison:** Participants evaluate which shape is bigger or smaller.



4. **Length Comparison:** Participants determine which segment is longer or shorter.

5. **Pathways Recognition:** Participants decide if water can pass through a given path, testing automatic visual pattern recognition.



Since the sets in each minigame appear just for one second, is not possible to rely on spatial attention shifts to identify pathways or objects (minigame 5,3), or to deliberate cognitive mechanisms as counting (minigames 1,2). WarmApp's emphasis on rapid numerical estimation and comparison is grounded in the concept of the Approximate Number System (ANS), a cognitive system that supports non-symbolic numerical representation. According to Coolen et al. (2018), ANS plays a crucial role in the development of mathematical abilities and is linked to intuitive and automatic cognitive processes. By engaging the ANS, WarmApp potentially activates brain areas associated with intuitive, System 1-type thinking -Lyu and Li (2019). Participants will repeat the Warmapp activity for 15 rounds. For each round, a score will eb calculated to meaasure the performance in that round. The scoring system in WarmApp is designed to reward accuracy and proximity to the correct answer, particularly in numerical estimation tasks. Answers to minigames 1,3,4 and 5 will provide 1 point if correct, 0 if wrong. Minigame 2 will provide 1 point if the distance between the answer and the actual number of items appeared is 2 or 3, and 2 points if it is 1 or 0. The average score of the first three rounds will be used as measure of System 1 proficiency. Starting from the third round, participants receive feedbacks on their performance, enhancing learning and engagement. This feedback is critical in reinforcing correct intuitive judgments and providing learning opportunities for incorrect responses. They will be informed whether their answers to minigames 1,3,4,5 were correct, and what were the actual number of items in minigame 2. Then, the rate of improvement will be measured at any round by taking the percentage change in the score. After the Warmapp activity participants will repeat the Ambiguity task outlined above.

4.3.4 Demographics, IQ, and Personality Assessment

The experiment concludes with a questionnaire gathering basic demographic information from participants, such as age and sex.

By including IQ and personality assessments with demographic data, we include potential drivers of risk and ambiguity, allowing for a precise evaluation of System 1 proficiency's role.

To assess cognitive ability, participants complete a short version of UNIT-IQ test consisting of 10 questions. An example question from the IQ test is shown below:

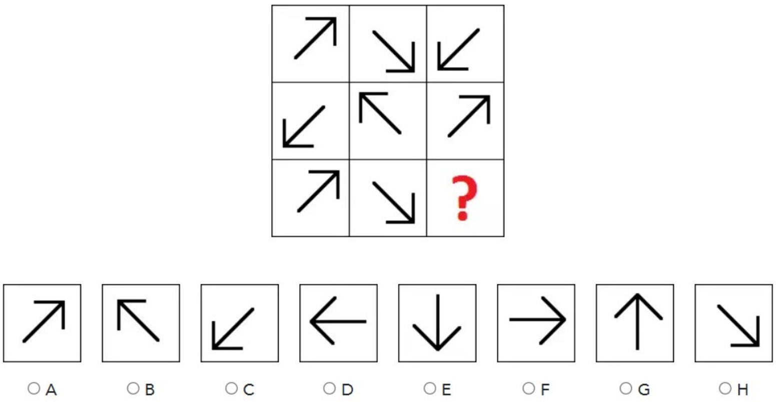


Figure 4.3: Example Question from UNIT Test

The inclusion of an IQ test is pivotal in this study as it allows us to control for cognitive abilities. Higher IQ might correlate with more effective information processing, potentially impacting risk and ambiguity aversion.

Personality traits are assessed using a short version of the Big Five personality test -Rammstedt and John (2007). The Big Five personality dimensions is a widely accepted model in psychological research, encompass five broad domains of personality traits. These dimensions are:

- **Openness to Experience:** This trait features characteristics such as imagination, insight, and a wide range of interests. Individuals high in Openness are typically curious and creative.
- **Conscientiousness:** This dimension is characterized by high levels of thoughtfulness, good impulse control, and goal-directed behaviors. Highly conscientious individuals are organized and mindful of details.
- **Extraversion:** Extraversion is marked by excitability, sociability, talkativeness, assertiveness, and high amounts of emotional expressiveness. Extroverted individuals are outgoing and often enjoy being around people.
- **Agreeableness:** This trait includes attributes like trust, altruism, kindness, affection, and other prosocial behaviors. People who score high in Agreeableness tend to be more cooperative.
- **Neuroticism:** This dimension refers to the tendency to experience unpleasant emotions easily, such as anger, anxiety, depression, or vulnerability. Neuroticism also refers to the degree of emotional stability and impulse control.

The questionnaire format and the method used to derive a measure for each personality dimension are illustrated below:

Instructions: How well do the following statements describe your personality?

I see myself as someone who …	Disagree strongly	Disagree a little	Neither agree nor disagree	Agree a little	Agree strongly
1. … is reserved	(1)	(2)	(3)	(4)	(5)
2. … is generally trusting	(1)	(2)	(3)	(4)	(5)
3. … tends to be lazy	(1)	(2)	(3)	(4)	(5)
4. … is relaxed, handles stress well	(1)	(2)	(3)	(4)	(5)
5. … has few artistic interests	(1)	(2)	(3)	(4)	(5)
6. … is outgoing, sociable	(1)	(2)	(3)	(4)	(5)
7. … tends to find fault with others	(1)	(2)	(3)	(4)	(5)
8. … does a thorough job	(1)	(2)	(3)	(4)	(5)
9. … gets nervous easily	(1)	(2)	(3)	(4)	(5)
10. … has an active imagination	(1)	(2)	(3)	(4)	(5)

Scoring the BFI-10 scales (R = item is reverse-scored):

Extraversion: 1R, 5

Agreeableness: 2, 7R

Conscientiousness: 3R, 8

Neuroticism: 4R, 9

Openness to Experience: 5R, 10

Figure 4.4: Big Five Personality Assessment

Understanding these personality dimensions is crucial in this study as they can significantly influence individual behavioral patterns, including decision-making processes in economic contexts. For instance, high Openness might correlate with a greater willingness to engage in new and uncertain economic ventures, while high Neuroticism could be linked to risk aversion.

At the end of the questionnaire, a random choice will be selected from the risk task and one from the ambiguity task. The lotteries selected by the participant in that choice are played and the payoff is determined. All participants receive the payoffs for this two lotteries and a show up fee of 5 euros.

4.4 Theoretical Framework

As explained in the previous sections, the tasks used to elicit ambiguity and risk attitutes involve a sequence of decisions between two options.

As in Holt and Laury (2002), in the risk aversion task each decision involves a choice between two lotteries (options) A and B. Each lottery has two possible outcomes: W_A, L_A in lottery A and W_B, L_B in lottery B. In both lotteries, the probability of the highest payoff (the outcome W_A in lottery A and W_B in lottery B) is p_k, and the lowest (L_A, L_B) is $(1-p_k)$, k being the decision round. The probability of the high payoff increases at every round k. The payoffs W_A, L_A, W_B, L_B are the same for every round, only the probability of the high payoff p_k increases at every round.

At any round we have $W_A > W_B$ and $L_A < L_B$, so that Option A has higher variance and increasing risk premium. The expected utility of individual i of each option in round k can be expressed as follows:

$$EU_{iAk} = p_k U_i(W_A) + (1-p_k)U_i(L_A)$$
$$EU_{iBk} = p_k U_i(W_B) + (1-p_k)U_i(L_B)$$

Where U represents the CRRA utility function:

$$U(x) = \begin{cases} \frac{x^{(1-r)}}{1-r} & \text{if } r \neq 1 \\ \ln(x) & \text{if } r = 1 \end{cases}$$

The participant switches from option A to B at the point where the expected utilities are equal. Hence, at the switching point, $EU_{Ak} = EU_{Bk}$. This condition provides us with an estimate of r, the coefficient of relative risk aversion:

$$p_k U_i(W_A) + (1-p_k)U_i(L_A) = p_k U_i(W_B) + (1-p_k)U_i(L_B)$$

The above defines an equation in the unknown r_i, whose solution will define the risk aversion parameter of the participant.

Now let's consider the sequence of Ellsberg's urns in the task for eliciting ambiguity attitues. Following the approach for the joint estimation of risk and ambiguity attitutes presented by Gneezy and List (2015), the utility of choosing the ambiguous urn can be represented as:

$$EU_{i,Ambiguous} = \alpha U_i(x_{min}) + (1-\alpha)U_i(x_{max})$$

Where x_{max} and x_{min} are respectively the highest and the lowest possible payoffs,α is the ambiguity aversion parameter, and U_i is the same CRRA utility function with parameter r_i estimated for each participant in the previous task.

In our case, both in the ambiguous and in the risky urn the possible payoffs are the same. Either a ball of the winning colour is extracted and the payoff is 5 (outcome W) or the payoff is zero (outcome L).

Consider q_j to be the winning probability in the risky urn at round j. At the switching point, $EU_{Risky} = EU_{Ambiguous}$, giving us:

$$q_j U_i(W) + (1 - q_j)U_i(L) = \alpha_i U_i(L) + (1 - \alpha_i)U_i(W)$$

By solving this equation α_i, which indicates the level of ambiguity aversion, is estimated.

If $\alpha_i > \frac{1}{2}$, the participant is ambiguity averse. If $\alpha_i = \frac{1}{2}$, the participant is ambiguity neutral. If $\alpha_i < \frac{1}{2}$, the participant is ambiguity seeking.

Given that in our case the possible payoffs are the same both in the risky and in the ambiguous urn, the above equation yields the same result independently from the parameter of risk aversion and the specification of the CRRA. Therefore, the ambiguity aversion parameter can be estimated adopting the simpler normalization $Ui(W) = 1$ and $Ui(L) = 0$, that, applied at the above equation in which $EU_{Risky} = EU_{Ambiguous}$ is imposed at the switching point, gives $\alpha_i = 1 - q_j$, where i indexes participants and j the choice at which the participant switched. This imply that the framework applied to our case is practically equivalent to separate estimation of risk and ambiguity aversion, therefore is necessary to conduct a simultaneous estimation in the empirical analysis.

4.5 Estimation

In this section we will describe the experimental dataset and the statistical methods used to assess whether individuals more proficient in system 1 thinking are more tolerant to risk and ambiguity and whether training System 1 proficiency ambiguity aversion can be reduced.

4.5.1 Data

The study commenced with a sample of 108 participants (recruited in five experimental sessions), divided into an experimental group of 83 and a control group of 25. The analysis focuses on those within the experimental group who engaged in the complete set of activities, including the WarmApp activity intended to assess and enhance System 1 proficiency. Data were subjected to a cleaning process to ensure consistency, particularly within responses to the Holt and Laury task for risk aversion and the Ellsberg's urns task for ambiguity aversion. In this process, special attention was directed towards identifying discrepancies indicative of random responses, misunderstandings, or inconsistencies, which include the phenomenon of multiple-switching behavior (MSB). MSB, characterized by erratic switching between options in a manner that contradicts rational choice theory, signals potential violations of foundational assumptions such as first-order stochastic dominance or transitivity Charness et al. (2013), Chew et al. (2023). The prevalent interpretation within the literature posits that MSB contradicts the foundational assumptions of rational choice theory. Charness et al. (2013) highlight the difficulty in rationalizing ”such inconsistent behavior under standard assumptions on preferences,” suggesting that MSB may reflect violations of fundamental economic principles such as first-order stochastic dominance or transitivity . In our study, MSB was observed in 8,5% of decisions in the risky task and 7% in the ambiguity task, a prevalence consistent with findings from previous studies, highlighting the ubiquity of MSB across different experimental settings Holt and Laury (2002).The total number of participants showing MSB was 7 in the risky rask and 6 in the ambiguity task - including 3 participants exhibiting MSB in both tasks - for a total of 10 participants. Chew et al. (2023) further refine the understanding of MSB by differentiating between ”regular” and ”irregular” patterns, where the latter, characterized by erratic or non-progressive choice patterns, provides a clear rationale for exclusion from analysis of participants that exhibit irregular MSB behavior. According to their distinction, regular MSB is characterized by participants initially choosing options on the left and eventually switching to options on the right, a pattern potentially reconcilable within certain theoretical frameworks. Conversely, irregular MSB—manifested by participants either persistently choosing options on the same side or initiating choices on the right and concluding on the left—signals a deviation from rational decision-making paradigms that is harder to justify within standard economic models. In our experiment, inconsistencies were specifically due to the presence of irregular MSB, a pattern that undermines the assumption of coherent preference ordering. This justifies exclusion in alignment with the broader empirical literature already cited, further supported by the literature's emphasis on data quality and the integrity of economic inference, ensuring that analyses are based on behavior that reflects coherent preference structures, such as List and Gallet (2007), Lusk and Schroeder (2005). Exclusion will refine our sample to 73 individuals for the experimental condition. However, to further ensure the robustness of our analysis and to explore the potential impact of multiple-switching behavior (MSB) on our findings, we conducted additional estimates that include participants exhibiting both regular and irregular MSB. Recognizing the complexity associated with MSB in risk and ambiguity aversion tasks, this supplementary analysis aims to provide a comprehensive view of how the inclusion of these participants might influence the overall interpretations of risk and ambiguity preferences, facilitating a deeper understanding of the sensitivity of our conclusions to the inclusion of participants with MSB. In addressing the complexities introduced by MSB in our dataset, we have adopted a methodological approach that involves averaging the switching points for participants exhibiting MSB. This decision is underpinned by the insights provided by Yu et al. (2021), who explore the phenomenon of MSB within the context of Multiple Price Lists (MPL) experiments. While Yu et al. acknowledge that MSB may arise from task miscomprehension, they also argue for the inclusion of these participants' data as a means to enhance the robustness of empirical analyses. By averaging switching points, we can integrate the full range of decision-making behaviors observed in our study, thereby adhering to a methodological framework that acknowledges the nuances of participant responses.

These additional estimates are presented in the Appendix.

Descriptive statistics for both the cleaned sample and the full sample are reported in Table 4.1, which categorizes participants based on their System 1 proficiency as measured by initial WarmApp activity scores. The table is divided into three columns for the whole sample, the High System 1 Proficiency group, and the Low System 1 Proficiency group, detailing the average of the relevant variables measured in the experiment.

Table 4.1: Descriptive Statistics for the Full Sample, Reduced Sample, High System 1 Proficiency, and Low System 1 Proficiency Groups

Variable	Full Sample	Reduced Sample (N=73)	High System 1	Low System 1
Number of Observations	83	73	61%	39%
Avg. Ambiguity Aversion	0.586	0.588	0.58	0.6
Avg. Risk Aversion	0.319	0.34	0.3	0.4
Percentage of Males	53.94%	49%	52%	44%
Avg. IQ	97.52	96.6	97	96
Avg. Openness	6.90	6.97	7.1	6.85
Avg. Conscientiousness	7.36	7.45	7.45	7.46
Avg. Extraversion	5.55	5.72	5.6	5.9
Avg. Agreeableness	5.71	5.72	5.6	5.9
Avg. Neuroticism	6.35	6.32	6.4	6.2
Avg. Rate of Improvement	0.154	0.15	0.18	0.11

Note: The full sample includes all participants, whereas the reduced sample excludes those with multiple-switching behavior. High and low System 1 proficiency groups are determined based on the Warmapp activity score.

From the above table we can see that System 1 Proficients display on average lower ambiguity and risk aversion, less impovement on the sequence of the rounds of the Warmapp activity, and higher score in the personality traits of Extraversion and Openness. Before analyzing the significance of the differences in risk and ambiguity attitudes, we need to check that this difference do not arise from the difference in other covariates.

So I will first present a general linear regression model to evaluate the impact of System 1 proficiency and the other covariates (IQ, personality traits and demographics) on the degree of ambiguity aversion.

4.5.2 Seemingly Unrelated Regression Analysis

To comprehensively assess the impact of System 1 proficiency, measured through Warmapp activity, on both risk and ambiguity aversion, we employed a Seemingly Unrelated Regression (SUR) model. This approach allows for the simultaneous estimation of the two dependent variables—risk and ambiguity aversion—accounting for the potential correlation of error terms across equations. This methodology is particularly apt for our analysis, given the theoretical expectation that the determinants of risk and ambiguity aversion, including System 1 proficiency, demographics, personality traits, and IQ, may exert influences that are correlated across these dimensions of decision-making.

The SUR model is specified as follows for risk and ambiguity aversion, respectively:

$$\text{Risk}_i = \beta_0 + \beta_1 \text{WarmappScore}_i + \beta_2 \text{PersTraits}_i + \beta_3 \text{IQ}_i + \beta_4 \text{Demogr}_i + \varepsilon_{i1} \tag{4.1}$$

$$\text{Ambiguity}_i = \gamma_0 + \gamma_1 \text{WarmappScore}_i + \gamma_2 \text{PersTraits}_i + \gamma_3 \text{IQ}_i + \gamma_4 \text{Demogr}_i + \varepsilon_{i2} \tag{4.2}$$

where ε_{i1} and ε_{i2} are the error terms for the risk and ambiguity equations, respectively, which are allowed to be correlated in the SUR estimation framework.

Our SUR analysis, presented in Tables 4.2 and 4.3, reveals significant insights into the determinants of risk and ambiguity aversion. Most notably, System 1 proficiency (Warmapp Score) exhibits a statistically significant negative relationship with risk aversion, suggesting that higher proficiency is associated with lower risk aversion. This relationship is robust and significant at the 0.05% level, highlighting the importance of intuitive decision-making processes in risk-related contexts. Conversely, the impact of System 1 proficiency on ambiguity aversion, while negative, does not reach conventional levels of statistical significance, indicating a more nuanced relationship in contexts of ambiguity.

Inclusion of MSB participants in an augmented sample and re-estimation of the SUR model provided additional insights, affirming the robustness of our findings across different sample compositions. The results of this extended analysis are detailed in the Appendix, underscoring our commitment to thoroughness and the robustness of our analytical approach.

Table 4.2: SUR Estimation Results for Risk Aversion

Variable	**Coefficient**	**P-value**
WarmApp Score	-0.1070*	0.005
Openness	0.0117	0.6017
Neuroticism	0.1945***	0.001
Conscientiousness	0.0230	0.3842
Agreeableness	-0.0022	0.9152
Extraversion	-0.0331	0.2509
IQ	0.0012	0.6294
Age	-0.0075	0.6066
Gender (Female = 1)	-0.0594	0.3952

Note: Standard errors are omitted for brevity. ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

Table 4.3: SUR Estimation Results for Ambiguity Aversion

Variable	**Coefficient**	**P-value**
WarmApp Score	-0.0122	0.11
Openness	0.0087	0.0521
Neuroticism	-0.0007	0.8578
Conscientiousness	-0.0081	0.1216
Agreeableness	0.0054	0.1798
Extraversion	0.0031	0.5878
IQ	-0.0007	0.1597
Age	0.0002	0.9529
Gender (Female = 1)	-0.0029	0.8327

Note: Standard errors are omitted for brevity. ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

Neuroticism is the dimension of personality associated with emotional instability, anxiety, moodiness, irritability, and sadness. Individuals who score high in neuroticism may be more prone to stress and negative emotional states. In the context of risk aversion, it is plausible that individuals with high levels of neuroticism could be more risk-averse. This is because such individuals may have a greater sensitivity to potential losses or negative outcomes, which aligns with the core aspects of risk aversion where the individual prefers to avoid losses rather than acquiring equivalent gains. Therefore, finding a significant association between neuroticism and risk aversion would make sense, as more neurotic individuals may be more concerned about the possibility of negative outcomes and thus choose safer options.

The observed low significance levels in the relationship between System 1 proficiency and ambiguity aversion may be attributable to the constraints imposed by our relatively small sample size. To address this issue, a power analysis was conducted - as showed in the next section - revealing that the sample size indeed might not be sufficient to detect small but potentially meaningful effects, especially in the context of ambiguity aversion. Moreover, tables 4.4 and 4.5 show the results of SUR estimation for the complete sample that includes participants with MSB.

Table 4.4: SUR Estimation Results for Risk Aversion (Full Sample)

Variable	**Coefficient**	**P-value**
WarmApp Score	-0.1300***	0.0005
Openness	0.0091	0.6686
Neuroticism	0.1862***	0.0001
Conscientiousness	0.0216	0.3881
Agreeableness	-0.0031	0.8795
Extraversion	-0.0122	0.6483
IQ	0.0010	0.7117
Age	-0.0146	0.3255
Gender (Female = 1)	-0.0677	0.3287

Table 4.5: SUR Estimation Results for Ambiguity Aversion (Full Sample)

Variable	**Coefficient**	**P-value**
WarmApp Score	-0.0138**	0.0421
Openness	0.0081	0.095
Neuroticism	-0.0014	0.6769
Conscientiousness	-0.0100	0.29
Agreeableness	0.0042	0.2609
Extraversion	0.0031	0.5337
IQ	-0.0008	0.1141
Age	7.258e-05	0.9788
Gender (Female = 1)	-0.0034	0.7900

The SUR estimation conducted with the full sample, which includes participants exhibiting multiple-switching behavior (MSB), not only confirms but also strengthens the findings from the reduced sample analysis. The comprehensive analysis underlines a significant relationship between System 1 proficiency and both risk and ambiguity aversion. The inclusion of MSB participants, who constitute approximately 8.5% and 7% of the participants in each task respectively and collectively represent around 12% (10 out of 83) of the whole sample, enhances the statistical power of our analysis. This augmentation addresses the previously observed low significance levels in the relationship between System 1 proficiency and ambiguity aversion in the reduced sample. The limited statistical power in the reduced sample analysis could have hindered the detection of significant effects, as will be outlined in the "Power Analysis" section.

4.5.3 T-test

We first conducted an independent samples t-test to compare the ambiguity attitudes of System 1 High and Low Proficiency Groups - as presented in Table 1. Given the data:

$$\begin{aligned}
\bar{X}_1 &= \text{Mean of System 1 High Proficiency} = 0.58 \\
\bar{X}_2 &= \text{Mean of System 2 Low Proficiency} = 0.6 \\
s_1 &= \text{Standard Deviation of High System 1 Proficients} = 0.0573 \\
s_2 &= \text{Standard Deviation of Low System 1 Proficients} = 0.039 \\
n_1 &= \text{Sample size of High System 1 Proficients} = 45 \\
n_2 &= \text{Sample size of Low System 1 Proficients} = 28
\end{aligned}$$

The formula for the t-statistic for two independent samples is:

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Substituting in the provided data:

$$t = \frac{0.58 - 0.6}{\sqrt{\frac{0.0573^2}{45} + \frac{0.065^2}{28}}}$$

This yields:

$$t \approx -1.37$$

The degrees of freedom (df) for this test are given by:

$$df = n_1 + n_2 - 2 = 71$$

Thus, with a computed t-statistic of approximately -1.37 and df of 71, referring to the well-known table on the values of the t-distribution we can conclude that the observed difference is significant at 10% level, but not at the conventional alpha=0.05 level. The t-test increases the power and show that the difference in System 1 Proficiency has a slightly significant impact in determining ambiguity attitudes, but still not at the standard level required for statistical significance.

4.5.4 Power Analysis

In order to confirm that the low significance of the effect of System 1 Proficiency on ambiguity in the reduced sample is due to the small sample size, we implemented power analysis to derive the sample size required for having the conventional level of significance alpha=0.05.

To determine the required sample size for conventional levels of power and significance (0.8 and 0.05) the general formula from Cohen (1988) can be applied:

$$n = \frac{\lambda}{f^2} \tag{4.3}$$

Where λ is a parameter that depends on the number of predictors in the regression model and the desired significance and power. Using the tables in Cohen (1988) we can find the value of λ for our analysis, $\lambda = 18.4$. Cohen's f^2 is indeed one commonly used measure of effect size defined as:

$$f^2 = \frac{R^2}{1 - R^2} \tag{4.4}$$

Given the R^2 value of 0.147 for our regression model with nine predictors, we first calculated Cohen's f^2 using the formula:

$$f^2 = \frac{R^2}{1 - R^2} = \frac{0.147}{1 - 0.147} \approx 0.172 \tag{4.5}$$

With Cohen's f^2 calculated, the required sample size:

$$n = \frac{18.4}{0.172} = \approx 107 \tag{4.6}$$

Concluding, the calculated sample size required for our multiple regression analysis, ensuring a power of 0.8 and a significance level of 0.05, is approximately 107 observations, which is larger than our sample.

The required sample size for a two-sample t-test can be calculated using the formula:

$$n = 2 \cdot \left(\frac{(Z_{\alpha/2} + Z_{\beta}) \cdot \sigma}{\delta} \right)^2$$

Given that:

- σ is the pooled standard deviation, calculated as $\sigma = \sqrt{\frac{(n_1-1)\cdot\sigma_1^2+(n_2-1)\cdot\sigma_2^2}{n_1+n_2-2}}$.
- δ is the difference in means, which is 0.02.
- n_1 and n_2 are the sample sizes of the two groups.

Let's consider the scenario where $\sigma_1 = 0.057$ (High System 1 Proficients), $\sigma_2 = 0.065$ (Low System 1 Proficients), $n_1 = 45$, and $n_2 = 28$. First, compute the pooled standard deviation:

$$\begin{aligned}
\sigma &= \sqrt{\frac{(45-1)\cdot 0.057^2 + (28-1)\cdot 0.065^2}{45+28-2}} \\
&= \sqrt{\frac{44\cdot 0.003249 + 27\cdot 0.004225}{71}} \\
&= \sqrt{\frac{0.143156 + 0.114075}{71}} \\
&\approx \sqrt{\frac{0.257231}{71}} \\
&\approx 0.0601
\end{aligned}$$

Now, plug in the values into the formula for the required sample size:

$$\begin{aligned}
n &= 2 \cdot \left(\frac{(1.96+0.84)\cdot 0.0601}{0.02} \right)^2 \\
&= 2 \cdot \left(\frac{2.80 \cdot 0.0601}{0.02} \right)^2 \\
&= 2 \cdot (8.4036)^2 \\
&\approx 2 \cdot 70.6202 \\
&\approx 141.24
\end{aligned}$$

So that a total sample size of approximately 141 is required for the t-test to detect a difference in means of 0.02 with 80% power and at a 5% significance level.

Power Analysis suggests that the low significance of the result in the reduces sample is probably due to the small sample size, and the SUR estimation conducted on the full sample is increasing significance due to the increase in power.

4.5.5 Wilcoxon Rank-Sum Test (Mann-Whitney U)

The Wilcoxon rank-sum test, also known as the Mann-Whitney U test, is a non-parametric method designed to test for differences between two independent samples on ordinal or continuous data. It does so by comparing the medians of the two distributions rather than their means. The test doesn't assume data normality and is particularly useful when dealing with non-normally distributed datasets. Given the size of our reduced sample, the small variability in our measure of ambiguity aversion and its distribution being non properly normal, this test is suited to give robustness to our analysis.

The primary steps in the Mann-Whitney U test are:

1. Combine all the data from the two groups and rank them.

2. In the case of tied ranks (i.e., identical values), assign the average of the ranks that would have been assigned had the values been slightly different.

3. Sum the ranks for each group separately.

4. Compute the U-statistic using the given formulae for each group.

5. The smallest U-statistic is the test statistic.

6. Under the null hypothesis, U follows a known distribution from which we can derive a p-value or a critical z-score to test the hypothesis.

In our study's context, we aimed to determine if there was a significant difference in ambiguity aversion between those more proficient on System 1 cognitive process.

Given the data:

- High System 1 Proficients: Sum of ranks = 132

- Low System 1 Proficients: Sum of ranks = 264

The U-statistics for both groups were computed as:

$$U_1 = n_1 \times n_2 + \frac{n_1(n_1+1)}{2} - \text{Rank sum of High System 1 Proficients}$$
$$U_1 = 45 \times 28 + \frac{45(45+1)}{2} - 132$$
$$U_1 = 2163$$

$$U_2 = n_1 \times n_2 + \frac{n_2(n_2+1)}{2} - \text{Rank sum of Low System 1 Proficients}$$
$$U_2 = 45 \times 28 + \frac{28(28+1)}{2} - 264$$
$$U_2 = 1402$$

Given U_2 is the smaller U-statistic, we use it for the test. To assess the significance, the U-statistic can be transformed to a z-score, especially if the sample size is sufficiently large. The expected mean of the U statistic, μ_U, is derived from the principle of equiprobability. Under the null hypothesis (where both samples come from the same population), each observation from one sample has an equal chance of being greater or lesser than an observation from the other sample. This equiprobability principle gives rise to the formula:

$$\mu_U = \frac{n_1 \times n_2}{2}$$

Here, n_1 and n_2 are the sample sizes of the two groups. This formula essentially states that, in the absence of any real difference between the groups, the expected number of times a value from the first group ranks above a value from the second group is exactly half of all possible comparisons.

The variance of the U statistic, σ_U^2, considers the spread of the ranks across all potential configurations of the combined groups. Its derivation is a bit more involved but is built on the combinatorial foundation of the test. Given the total number of possible rank combinations, the variance is calculated as:

$$\sigma_U^2 = \frac{n_1 \times n_2 \times (n_1 + n_2 + 1)}{12}$$

This formula reflects the dispersion or spread of ranks that could arise by chance alone when two groups of sizes n_1 and n_2 are drawn from the same population. The factor of $\frac{1}{12}$ is a result of combinatorial calculations and integrates the possible spread in the rankings for the two groups (Mann and Whitney (1947), Wilcoxon (1945)).

$$\mu_U = \frac{n_1 \times n_2}{2}$$
$$\mu_U = \frac{45 \times 28}{2} = 630$$

$$\sigma_U = \sqrt{\frac{n_1 \times n_2 \times (n_1 + n_2 + 1)}{12}}$$
$$\sigma_U = \sqrt{\frac{45 \times 28 \times (45 + 28 + 1)}{12}} \approx 189.3$$

The z-score is then:

$$z = \frac{U_2 - \mu_U}{\sigma_U} = \frac{1402 - 630}{189.3} \approx 4.07$$

By standardizing the U statistic using its expected mean and variance (essentially converting it into a z-score), we can gauge the extremeness of our observed result in terms of standard deviations from the mean. is our observed U, given what we would expect if there were no difference between the groups?" Our calculated z-score of 4.07 is significantly distant from zero, the expected value under the null hypothesis. In standard statistical practice, a z-score beyond ± 1.96 (representing the 5% level of significance for a two-tailed test) is considered significant. Our result far exceeds this, and referring to the well-known table on the values of the z-distribution we can conclude that the observed difference is significant at 1% level. This result emphasize that the observed differences in ambiguity aversion between High System 1 Proficients and Low System 1 Proficients are not likely due to random fluctuations but represent a substantive effect, that the previous methods were not able to capture with the required level of significance due to the small sample size, as showed in the Power Test section.

4.5.6 Impact of System 1 Training on Ambiguity Aversion

This section examines the relationship between System 1 training and changes in ambiguity aversion among individuals classified as low in System 1 proficiency. The empirical strategy exploits the variance in performance improvements over the course of 15 rounds of the WarmApp activity, following learning by doing, feedbacks and nudges designed to enhance intuitive decision-making. The rate of improvement in the score is measured for any round, and then an average rate of improvement is calculated across all the 15 rounds to smooth these difference in performances an capture a general measure of the improvement in system 1 proficiency during the training.

A regression analysis was conducted where the dependent variable was the change in ambiguity aversion measured before and after the intervention. The key independent variable of interest is the average rate of improvement in WarmApp scores for individuals initially identified as having low System 1 proficiency. The model controls for various demographic characteristics, including age and sex, as well as cognitive ability (IQ) and personality traits, in accordance with the Big Five personality model.

The following regression equation was estimated:

$$\Delta\alpha_i = \beta_0 + \beta_1 \hat{r}_i + \beta_2 \text{PersTraits}_i + \beta_3 \text{IQ}_i + \beta_4 \text{Demogr.}_i + \varepsilon_i \tag{4.7}$$

where $\Delta\alpha_i$ represents the change in ambiguity aversion score for individual i, $\hat{r}_i$ is the average rate of improvement in WarmApp scores, $\text{PersonalityTraits}_i$ includes the Big Five personality dimensions and Demogr._i includes age and sex.

The results are presented in Table 4.6. The coefficient on the average rate of improvement (β_1) is negative and statistically significant, suggesting that individuals with lower initial System 1 proficiency can reduce their ambiguity aversion through targeted training. The effect size is robust when accounting for other personality traits and demographic factors.

Table 4.6: Regression Analysis on Change in Ambiguity Aversion

Variable	Coefficient (Standard Error)	p-value
Constant	-0.0.95 (0.059)	0.11
Average Rate of Improvement	-0.05** (0.03)	0.06
Openness	-0.004 (0.002)	0.04
Neuroticism	-0.0015 (0.002)	0.44
Conscientiousness	0.0001 (0.002)	0.9
Agreeableness	-0.003 (0.002)	0.11
Extraversion	-0.001 (0.003)	0.57
IQ	0.0005 (0.002)	0.85
Age	-0.0013 (0.001)	0.54
Sex (Female = 1)	-0.04 (0.07)	0.54
Observations	28	
R-squared	0.40	

Note: Standard errors are in parentheses. p-values correspond to two-tailed tests.

Table 4.7 shows indeed the results of the same estimation conducted in the full sample, including participants with MSB.

Table 4.7: Regression Analysis on Change in Ambiguity Aversion including MSB partipants

Variable	Coefficient (Standard Error)	p-value
Constant	-0.046 (0.052)	0.38
Average Rate of Improvement	-0.06*** (0.02)	0.01
Openness	-0.003 (0.002)	0.14
Neuroticism	-0.001 (0.001)	0.58
Conscientiousness	0.002 (0.002)	0.39
Agreeableness	-0.003 (0.002)	0.16
Extraversion	0.0001 (0.002)	0.96
IQ	0.0002 (0.006)	0.99
Age	-0.001 (0.001)	0.41
Sex (Female = 1)	-0.05 (0.07)	0.48
Observations	28	
R-squared	0.40	

Note: Standard errors are in parentheses. p-values correspond to two-tailed tests.

Again, including MSB partipants enhances power and confirms and strenghten the results.

To further ensure the robustness of our findings, we evaluated the control group, which did not participate in the WarmApp training, together with the whole sample, to ascertain that the change in ambiguity aversion is not a result of time or external factors. As indicated in Table 4.8, there was no significant general change in ambiguity aversion in the control group nor in the whole sample, further validating the effectiveness of the training program for the treatment group.

Table 4.8: Robustness Check: Whole Sample and Control Group

Average Change in Ambiguity Aversion (Whole Sample)	0.01
Average Change in Ambiguity Aversion (Control Group)	0.01

In conclusion, the evidence supports the hypothesis that System 1 training, via repeated engagement with the WarmApp activity, significantly reduces ambiguity aversion in individuals with initially low System 1 proficiency. This finding has important implications for the field of behavioral economics, suggesting that cognitive training can play a role in enhancing decision-making processes that are more resilient to ambiguity.

4.6 Discussion and Conclusion

This study contributes to the field of behavioral economics by shedding light on the role of System 1 proficiency in decision-making under conditions of risk and ambiguity. Our methodological approach, which began with assessing participants' risk and ambiguity aversion and continued with the WarmApp activity, has provided key insights.

We observed a strong and statistically significant correlation between high System 1 proficiency and increased risk tolerance. This finding underlines a robust link between intuitive cognitive processing and risk-taking behavior, confirming previous studies. The implications of this are relevant especially in understanding decision-making in risk-laden scenarios.

Regarding ambiguity, while the relationship between System 1 proficiency and ambiguity aversion was less pronounced in the SUR estimation with the sample cleaned of MSB participants, after incorporating MSB participants -thus increasing the statistical power- the relationship between System 1 proficiency and ambiguity aversion remains negative, as hypothesized, and reaches statistical significance at the 0.05 level. This outcome indicates that, while the connection between System 1 proficiency and ambiguity aversion appeared subdued in the initial SUR estimation from the sample excluding MSB participants, further analysis and power test suggest that the initial insufficiency in significance can be ascribed to limitations in sample size. The Wilcoxon test, applied to examine differences in ambiguity aversion, particularly between groups with varying levels of System 1 proficiency, revealed highly significant findings. This suggests that intuitive processing plays a significant role, a revelation that is not as apparent in traditional regression models but becomes evident when comparing group differences.

Moreover, participants with initially low System 1 proficiency showed improvement in ambiguity aversion after WarmApp training. Although marginally significant (p-value 0.09), this improvement was robust against controls on the control group and the whole sample, suggesting that targeted System 1 training, as facilitated by the WarmApp, can effectively reduce ambiguity aversion, particularly in individuals with lower baseline System 1 proficiency. These results not only advance our understanding of the cognitive underpinnings of economic behaviors but also open new avenues for research and applications.

These results have substantial implications across various domains. In mathematics education, for instance, they highlight the importance of encouraging risk-taking and exploratory learning approaches, resonating with the works of Sharma (2015) and Lake (2019). In fields like finance and electoral decision-making, our findings suggest that enhancing System 1 proficiency can significantly impact economic and political choices.

Looking ahead, one interesting research extension is the possibility of a longitudinal study, starting from primary school, involving prolonged WarmApp intervention with long-term monitoring. Such a study could assess the impact of sustained training on intuitive cognitive abilities over an extended period, providing invaluable insights into the development of decision-making skills and their long-term influence on risk and ambiguity attitudes.

In sum, this study enhances our understanding of the cognitive mechanisms underpinning economic behaviors under risk and ambiguity and opens up new pathways for future research and practical applications especially related to education. By highlighting the significant role of intuitive cognition in shaping attitudes towards risk and ambiguity and its potential enhancement through training, our research offers valuable insights for developing strategies to improve decision-making skills across various domains.

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